The rotation curve of spiral galaxies and its cosmological implications

E. Battaner & E. Florido
Dpto. Física Teórica y del Cosmos. Universidad de Granada. Spain

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Abstract

We review the topic of rotation curves of spiral galaxies emphasizing the standard interpretation as evidence for the existence of dark matter halos. Galaxies other than spirals and late-type dwarfs may also possess great amounts of dark matter, and therefore ellipticals, dwarf spirals, lenticulars and polar ring galaxies are also considered. Furthermore, other methods for determining galactic dark matter, such as those provided by binaries, satellites or globular clusters, have to be included. Cold dark matter hierarchical models constitute the standard way to explain rotation curves, and thus the problem becomes just one aspect of a more general theory explaining structure and galaxy formation. Alternative theories also are included. In the magnetic model, rotation curves could also be a particular aspect of the whole history of cosmic magnetism during different epochs of the Universe. Modifications of Newtonian Dynamics provide another interesting possibility which is discussed here.

Keywords: rotation curve, spiral galaxies, cosmology.

Contents

1 Introduction

There is a wide consensus that the rotation curves of spiral galaxies constitute an observational proof -perhaps the best proof- for the existence of dark matter in the Universe. Dark matter is of evident interest in Cosmology, hence the interest of a review on the topic, in this case from a post-graduate didactic point of view.

Dark matter is not an exotic or sophisticated hypothesis. Neutrinos, brown dwarfs and black holes are all candidates to be identified with dark matter and are, nonetheless, classical concepts in Physics, introduced by fully established theories. High Energy Physics actually predicts a number of particles that do not interact with photons. We cannot claim that all the existing matter emits or absorbs photons.

However, although the necessity of dark matter was proposed more than 60 years ago (Zwicky, 1937), this hypothesis is still not whole-heartedly accepted by some workers. This scepticism is never explicitly expressed, but is subjacent, implicitly revealed in sentences such as "a galaxy *has* a halo". We should rather say "a galaxy *is* a halo" as a galaxy's mass may be at least 10 times its visible mass, the visible element then being a mere minor component, only important for us because it is what we see.

(We must, however, accept that when we state that if a galaxy *is* a halo, the discovery of just one exception, i.e. of a visible galaxy with no halo, would represent a serious problem of interpretation, a problem as fundamental as finding the light of a firefly not associated with a firefly.)

Once the existence of dark matter is recognized as a conservative possibility, let us establish a difference between the problem of dark matter in the Universe and the problem of dark matter in galaxies. We implicitly assume throughout the paper that $\Omega \sim 1$, and that visible matter only contributes with $\Omega_V \sim 0.003$. The total matter contributes either with $\Omega_M \sim 1$, as classically assumed, or with $\Omega_M \sim 0.3$, coherent with the more recently assumed scenario deduced from the observations of early supernovae, leading to the re-accelerating Universe, the non-vanishing cosmological constant or other identifications of dark energy (e.g. Turner, 1999). The assumption of a plane Universe, $\Omega \sim 1$, is based on theoretical ideas about inflation, observations of large-scale dynamics, the interpretation of the CMB and even on some philosophical arguments. Our position on dark matter is based on the difference between $\Omega_M \sim 0.3$ and $\Omega_V \sim 0.003$, as observed. We only see, in the best of cases, 1% of the matter. This is the basic fact, rather than the existence and possible observation of dark matter in galaxies, that demonstrates that dark matter exists.

Rotation curves and other dynamical effects in galaxies suggest that total galactic mass is about 10 times larger than that observed. Even in this case, we should still find about 90% of dark matter elsewhere in the Universe, which means, given the relative uncertainty in all these figures, that practically all the required dark matter lies outside galaxies. The possible galactic contribution to dark matter is negligible to close the Universe. Therefore, the existence of galactic dark matter is clearly very important for our knowledge of galaxies and their dynamics, but not so decisive for the cosmological problem of identifying its nature and amount.

This notion allows us to make a more objective analysis of the topic of dark matter in galaxies, once it has been partially disconnected from the cosmological one. Rotation curves of spirals, and many other observations, are currently interpreted as evidence of the existence of massive large dark halos, but a critical analysis of the observations, and the theoretical interpretations involved, is permanently necessary. Even if dark matter is a major ingredient in the most widely accepted and orthodox picture of a galaxy, most authors only differing about its mass and size, we will see that the hypothesis of a total absence of galactic dark matter cannot be completely ruled out.

The problems involved in determining galactic dark matter may be summarized as follows: the internal regions of galaxies require little or no dark matter and we must examine the external ones, where there are few stars and we must observe gas to determine the gravitational potential. However gas, in these regions where the gravitational force is low, may be influenced by magnetic fields. Thus, it is convenient to look for stellar systems lying far from the galaxy, and particularly satellite or companion galaxies. Then, however, it is very difficult to distinguish between galaxies with a halo and halo with galaxies, i.e. the hypothesis of a large common halo is difficult to reject.

Though the rotation curves of spirals is the main topic to be discussed, we must pay attention to other methods of determining DM, in particular those based on globular clusters, satellite galaxies, binary systems, polar-ring galaxies and so on. Lensing by spiral galaxies could become an important tool in the near future (Maller et al. 1999). We should also review the methods of DM estimation in other types of galaxies. With respect to the DM problem, spirals do not seem to be exceptional.

Other models, which do not require DM to explain rotation curves, have been reported in the literature and also require our attention in this study. One such model is MOND (Modified Newtonian Dynamics) and another is the magnetic model. The authors have contributed to the development of this latter model, which is commented in some detail at the end, together with its cosmological implications. Despite the inclusion of MOND and the magnetic approach as interesting possibilities, through out this paper we adhere to the most conservative point of view based on DM. Nevertheless, we emphasize the difficulties inherent to most methods and models.

The purpose of this review is mainly didactic, as a bibliographic source for postgraduate courses, but also critical. This critical approach might be considered unnecessary, in view of the wide acceptance of the dark matter hypothesis by the scientific community; but apart from its didactic interest, it is always pertinent to reconsider apparently solid beliefs. In this respect, it is convenient to remind the reader of four historical aspects related to the early adoption of the DM halo hypothesis, which were based on arguments that eventually became open to discussion or were made obsolete:

a) Kahn and Woltjer (1959) considered the dynamics of the double system formed by M31 and the Milky Way and concluded that their motion would require a binary system mass of $\geq 1.8 \times 10^{12} M_{\odot}$, much greater than the visible matter in the two galaxies. Kahn and Woltjer formed the opinion that there existed an as yet unobserved mass in some invisible form. Although they identified this invisible mass with hot gas rather than dark matter in its present sense (unobserved rather than unobservable) this work gave a first proof of the missing mass. It should be remembered, however, that this mass was considered to lie either in M31 and the Milky Way or in the intergalactic space (either two halos or a common halo). It is still not clear whether this second possibility can be completely ruled out. Therefore, this paper established the existence of dark matter, but not necessarily within galaxies. The doubt remained: either in or in between.

b) Oort (1960, 1965) found evidence for the presence of dark matter in the disk of our galaxy, although van dem Bergh (1999) wrote: "However, late in his life, Jan Oort told me that the existence of missing mass in the galactic plane was never one of his most firmly held scientific beliefs". After a long debate since then (see Binney and Tremaine, 1987; Ashman 1992) the discussion finally seems to be closed. Using HIPPARCOS data, Crezé et al. (1998) have found that there is no evidence for dark matter in the disk: gas and stars perfectly account for the gravitational potential.

c) Ostriker and Peebles (1973) suggested that spherical dark matter halos around the visible component of the spiral galaxies were necessary to suppress bar instabilities. However, their arguments did not convince Kalnajs (1983) and Sellwood (1985), who showed that a central bulge was equally efficient to stabilize disks.

d) Babcock (1939) observed that the stars in M31 were rotating at an unexpectedly high velocity, indicating a high outer mass-to-light ratio, although he also considered other possibilities: either strong dust absorption or, as he stated, "new dynamical considerations are required, which will permit of a smaller relative mass in the outer parts". This sentence, quoted by van dem Bergh (1999), is interesting, bearing in mind that gravitation is the only force considered at present to explain rotation curves, while other "dynamical considerations" are ignored. Optical rotation curves of other galaxies were obtained, until those published by Rubin et al. (1980), that were considered to be clear evidence of dark matter in galaxies. Today, however, many authors consider that optical rotation curves can be explained without dark matter (even without rejecting its contribution). For instance, Broeils and Courteau (1997) by means of r-band photometry and H_α rotation curves for a sample of 290 spirals concluded that "no dark halo is needed".

Therefore, the basic initial arguments leading to the belief of dark matter halos around spiral galaxies failed or were not conclusive. Only the interpretation of the HI rotation curves by Bosma (1978) subsists unmodified. For early thoughts on dark matter, the paper by van dem Bergh (1999) and the Ph.D. Thesis of Broeils (1992) are essential reading.

In brief, there is a standard interpretation of the rotation curve of spiral galaxies that is implicitly adopted throughout this paper, but we should not ignore other possibilities.

2 Rotation curves of spiral galaxies and dark matter

At large distances from the galactic centre the gravitational potential should be that produced by a central point mass and, in the absence of forces other than gravitation, it should be expected that $GM/R^2 = \theta^2/R$ (G, universal gravitation constant; M, galactic mass; R, galactocentric radius; θ, rotation velocity), therefore $\theta \propto R^{-1/2}$, which is called, for obvious reasons, the Keplerian rotation curve. This Keplerian decline was not observed, but rather, flat rotation curves with θ=cte were obtained. Apparently, this has the direct implication that $M \propto R$, thus depending on the quality of the telescope used. The "Dark Matter" (DM) hypothesis interprets this result in the sense that the Keplerian regime holds at much greater distances than those at which we obtain observations. There should be great quantities of dark matter extending far beyond the visible matter in a more or less spherically symmetric DM halo. If its distribution is spherically symmetric, the mass interior to a sphere of radius R would be $M(R) \propto R$, so that we obtain a first rough model of DM density distribution: $\rho = (1/4\pi R^2)dM/dR = \theta/4\pi GR^2$, i.e. $\rho \propto R^{-2}$, for distances far beyond the visible radius. This model is obviously over simplified, as we will see, but it coincides with the so called "nonsingular isothermal" profile

$$\rho = \frac{\rho_0}{1 + \left(\frac{R}{R_0}\right)^2} \tag{1}$$

(with ρ_0 and R_0 being constants), one of the most frequently types of halos.

The interpretation of rotation curves of spiral galaxies as evidence of DM halos was probably first proposed by Freeman (1970) who noticed that the expected Keplerian decline was not present in NGC 300 and M33, and considered an undetected mass, with a different distribution for the visible mass. The observation of flat rotation curves was later confirmed and the DM hypothesis reinforced by successive studies. Rubin, Ford and Thonnard (1980) and Bosma (1978, 1981a,b) carried out an extensive study, after which the existence of DM in spiral galaxies was widely accepted. Van Albada et al. (1985) analyzed the rotation of NGC 3198 and the distribution of its hypothetical DM, concluding that this galaxy has a dark halo, in agreement with the paper by Ostriker and Peebles (1987) about the stability of disks. The rotation of spirals was soon considered the most solid proof for the existence of DM in the Universe, particularly important when it was later believed that $\Omega = 1$. Other decisive papers were produced by Begeman (1987) and Broeils (1992).

The initial conclusion could be schematized by considering the rotation curve to be high and flat. If it is high, the dark halo should be very massive; if it is flat, the dark halo should be very large. Indeed, the flatness of the rotation curves was explained "too" well, because if the disk and halo had such different distributions, very careful matching was required between the falling disk rotation curve and the rising halo one. The curve was "too" flat; there was a "disk-halo conspiracy" (Bahcall and Casertano, 1985, van Albada and Sancisi, 1986).

The only explanation offered for this "conspiracy" is the adiabatic compression of the halo material when the disk was formed (Barnes 1987, Blumenthal et al. 1986) (which is commented later) although Bosma (1998) gave a list of galaxies for which this mechanism is not fully operational. The disk-halo conspiracy is a problem that remains to be satisfactorily solved. The problem is not why curves are flat (not all are flat) but why the transition from disk to halo domination is so smooth.

Different procedures have been used to obtain dark matter distribution: stellar distribution is determined from photometric observations and must then be complemented with CO and HI observations (with a correction factor to include the He mass) mainly for late spirals, in order to assess the gas profiles. These data determine the densities of bulge, disk and gas in the disk, or rather their contribution to the rotation velocity through the so called "circular velocity", $V_c(R)$, which would coincide with the true rotation velocity θ, if the component were dominant in the galaxy. The rotation curve, $\theta(R)$, is determined mainly with 21 cm maps. The addition of the different visible components does not, in general, coincide with $\theta(R)$, from which the existence of a DM halo is deduced.

Then, to obtain its distribution, there are several different techniques. One of the most widely used is the "maximum disk hypothesis" (see for instance, Begeman, Broeils and Sanders, 1991). Here, the mass to light, M/L, ratio is fixed for the different visible components, with values higher than about 10 being difficult to assign to a stellar population. Then, the innermost regions are adjusted so that the disk is able to produce the observed rotation curve without a halo. The disk M/L obtained is then kept constant at all radii and the circular velocity of the halo is then obtained for higher radii. Another possibility is the so called "best fit" technique. In this case, it is necessary to adopt a halo profile defined with several adjustable parameters. Most decompositions have adopted the isothermal spherical profile. At present, it might be profitable to investigate the alternative NFW profile, as this has a higher theoretical justification (we will come back to this point in the section devoted to theoretical models). The problem with the best fit procedure is that the halo distribution function must be known although, in part, this is precisely what we want to obtain.

The maximum disk technique was introduced by van Albada and Sancisi (1986). There are some psychological aspects to their introduction: "dark matter is a daring assumption; the intention is therefore to make the halo as small as possible, at least in the traditional optical best known innermost regions and reserve the exotic physics for the outer radio observable regions. It can therefore be found as noticeable that "maximum disk" fits are reasonable and do not very much differ from other fits. This gives us a first information: the inner parts do not require large amounts of dark matter". This conclusion was "a priori" not obvious. At present, it is considered that the amounts of dark and visible matter in the optical disks are similar, with not so much DM being needed as in the outermost disks.

The basic initial description consisting of an innermost region in which $\theta(R)$ increased linearly followed by a constant θ in the outer region was soon considered too simple. Casertano and van Gorkom (1991) found galaxies with declining rotation curves and analyzed current observations to show that bright compact galaxies have slightly declining rotation curves and that rising curves are predominant in low-luminosity galaxies (see also Broeils, 1992). This latter fact indicated that low-luminosity galaxies are more DM rich and that, in general, there is an increase in the dark matter fraction with decreasing luminosity (Persic and Salucci, 1988, 1990). Nearly all rotation curves belonging to the different types of spirals can be described by means of a single function, the so called "Universal Rotation Curve" (Persic, Salucci and Stel, 1996; Salucci and Persic, 1997) which is a successful fit of galactic astronomy that will be commented later.

2.1 Some examples of typical rotation curves

Figures 1, from the PhD Thesis of Begeman (1987), illustrates the photometric profiles used to obtain the bulge and disk contributions to the total rotation velocity, the observational rotation curve and the circular velocity of the halo. This is calculated from

$$\theta = [V_{gas}^2 + V_{disk}^2 + V_{bulge}^2 + V_{halo}^2]^{1/2} \tag{2}$$

where the V's are the circular velocities of the different components. With the "maximum disk" hypothesis we previously estimate V_{disk}^2 (in reality, also V_{bulge}^2), which determines M/L. It is usual to assume that the disk is exponential; then the disk circular velocity is calculated with the formula deduced by Casertano (1983) and that of the bulge with the formula given by Kent (1986). If the halo is spherically symmetric its circular velocity is simply $V_{halo}^2 = GM(R)/R$.

Note that the bulge, stellar disk and gas disk are insufficient to give the observed $\theta(R)$, and therefore either dark matter is needed or forces other than gravity are involved. Note also that, even if the maximum disk method gives reasonable results and real disks are probably near maximum disks, we assume $V_{halo} = 0$ in the central region, which implies $\rho_{halo} = 0$ in the central region, which is, from the point of view of galaxy formation, highly improbable. This fact, however, does not in practice introduce a serious problem.

On the other hand, we reproduce the rotation curve of the Sd galaxy NGC 1560 from the Broeils thesis. Figure 2 shows the obtention of the rotation curve from a velocity-position map. It is apparent that the curve is always an increasing function with no sign of becoming flat, which is typical of small galaxies. Fig 3 gives the contribution of the different components, following three methods, the "maximum disk", the "best fit" and the "minimum disk". This latter method is also called the "maximum halo", with a fixed M/L of only 0.1 (measured in solar units, $M_{\odot}/L_{\odot} = 1$ for the stellar disk, so that the disk gas makes the main contribution to the observable mass. This could be a reasonable assumption, as dwarf galaxies may be dominated by non-luminous material even in the innermost region. The "best fit" method again assumes a spherical isothermal halo.

2.2 Some problems

Direct inspection of the input data reveals some inherent problems in data handling. One is that this type of analysis is made under the assumption that disks are exponential. This "hypothesis" is reasonable as a zeroth order description, but there is an immoderate use of it in the literature. In Fig. 4 we see several photometric profiles of two galaxies (Begeman 1987). Rather than being an exception, the case of NGC 5033 is fairly typical. An extrapolation is needed in the bulge region (about 1 arcmin), there is a ring (about 1 arcmin) and a truncation at large radii (about 1 arcmin) and the stellar disk is less than 6 arcmin in radius.

The truncation of the disk (van der Kruit, 1979; van der Kruit and Searle, 1981a,b,1982a,b; Barteldrees and Dettmar, 1994) is very noticeable, for instance, in the galaxy NGC 5033. In Fig. 4 we plot the photometric surface brightness (in mag arcsec^{-2}) of this galaxy, showing a spectacular truncation, and the decomposition of the rotation curve following Begeman (1987). Is this disk really exponential?

Another problem may arise from the fact that the rotation curve is usually measured at 21 cm but the stellar disk in the optical. The stellar disk and the gas in the disk usually corotate, but due to frequent mergers and the accretion of clouds, captures, etc, this is not always the case. Unfortunately, non-corotation is more frequent than is generally assumed and very often the rotation curve of stars and of the gas differ greatly. Figures 5 and 6 are two examples taken from the Ph. D. thesis of Vega-Beltran (1997). In NGC 3898 the ionized gas and the stars not only counterrotate, but the shapes of the two rotation curves are quite different. The spiral galaxy IC 4889 is another surprising example: gas and stars do not counterrotate but the gas exhibits a typical flat curve while the stars decline in a Keplerian-like fall-off. These two examples are not an exception: important deviations from corotation are found in about 14 out of 22 galaxies in the Vega-Beltran sample, where gas and star rotation curves were measured independently.

There is another problem raised by Sofue and collaborators (Sofue, 1999; Sofue et al. 1999). These authors made a detailed measurement of the innermost rotation curve based on millimeter CO observations, because the frequent

HI hole in many galaxies impedes a proper observation using 21 cm. Their rotation curves have a steeper central increase, followed by a broad maximum in the disk and the characteristic flat rotation due to the massive halo. As the interest of rotation curves lies traditionally on the periphery, the central region has been neglected. From the observational point of view, this region is particularly difficult to study, specially in edge-on galaxies; a novel technique, the so called “envelope-tracing” method, has been used, in contrast with other current methods. Its logarithmic rotation curve of the Milky Way is reproduced in figure 7 and compared with the linear one of Clemens (1985) and Honma and Sofue (1997). The inner curve is of Keplerian type due to a central black hole. Observe that the curve should begin at the origin, $\theta(0) = 0$, but this very central steep increase has not yet been observed. Other galaxies have been observed to have a central fast rotation as in the Milky Way. These indicate the existence of dark matter in the centre, probably that of a black hole, which is also important in Cosmology, but the processes involved are different from those affecting the problems considered in this paper. Sofue et al. present rotation curves of 50 spirals, with a steep central rise, warning that previous curves could be incorrect in the centres, with the outermost regions remaining unaltered.

It is necessary to study how these results modify the standard methods of interpreting rotation data. For example, if the inner part of the rotation curve is steeper (Swaters, Madore and Trewhella, 2000), then M/L is understimated and, when using the maximum technique the luminous matter contribution is also understimated.

2.3 The Bosma relation

Bosma (1978, 1981b) and Carignan et al. (1990) found a trend for the gas distribution to have the same shape as DM distribution. This correlation between gas and DM is puzzling and if real, has no easy explanation in the light of present CDM models. Not only is there a general trend, but several individual features found in the rotation curve seems to correspond to features in gas circular velocity. This can be observed in Fig. 2 for NGC 1560 and Fig. 8 for NGC 2460.

This fact has inspired a theory (commented in section 3.1), identifying the dark matter with an as yet undetected dark gas. The magnetic hypothesis would provide another explanation, as the rotation curve is due in part to magnetic fields, which are generated by gas. A direct relation is not obvious when very extended curves are obtained (Corbelli and Salucci, 1999). Bosma (1998) himself states that this relation may not be correct.

2.4 The “Universal rotation curves”

Persic, Salucci and Stel (1996) and Salucci and Persic (1997) have analyzed a large number of rotation curves mainly catalogued by Persic and Salucci (1995) taking into account the $H\alpha$ data published by Mathewson, Ford and Buchhorn (1992) and also adopting some radio rotation curves. They claim that rotation curves can be fitted to what they call the "universal rotation curve" not only for any luminosity, but also for any type of galaxy, including spirals, low-surface-brightness, ellipticals and dwarf irregular galaxies. The existence of a "universal" rotation curve had previously been claimed by Rubin et al. (1980). Let us then reproduce the formulae of these "universal rotation curves", or PSS curves, restricting ourselves to spirals.

Following Persic and Salucci (1997), rotation curves of spirals can be fitted by a combination of two components. The first is an exponential thin disk, whose circular velocity can be approximated in the range $0.04R_{opt} < R \leq 2R_{opt}$ as

$$V_{disk}^2 = V^2(R_{opt})\beta \frac{1.97x^{1.22}}{(x^2+0.78^2)^{1.43}} \tag{3}$$

where x is a radial variable taking R_{opt} as unit

$$x = \frac{R}{R_{opt}} \tag{4}$$

R_{opt} is the radius encircling 83% of the light; for an exponential disk $R_{opt} = 3.2R_D$ where R_D is the radial scale length. β is a constant that depends on the luminosity. This function V_{disk} does not give a Keplerian fall-off for $x \to \infty$, nor is it the general expression of V_{disk} for exponential disks, but its application is restricted to a radial range.

The other component is the halo, with a circular velocity expressed as

$$V_{halo}^2 = V^2(R_{opt})(1-\beta)(1+a^2)\frac{x^2}{x^2+a^2} \tag{5}$$

where a is another constant, also depending on the luminosity. Then, the PSS curve is given by

$$V = \left(V_{halo}^2 + V_{disk}^2\right)^{1/2} \tag{6}$$

the contribution of a bulge therefore being considered negligible. The constants a and β are functions of the galaxy's luminosity, the best results being obtained for

$$\beta = 0.72 + 0.44 \lg \frac{L}{L_*} \tag{7}$$

$$a = 1.5\left(\frac{L}{L_*}\right)^{1/5} \tag{8}$$

where $L_* = 10^{10.4}L_\odot$. Then for a galaxy with luminosity L_*, a corresponds to a value of x of the order of R_{opt}, exactly $1.5R_{opt}$. Note that these values provide good fits, but for $L > 4.33L_*$ give negative values of V_{halo}^2.

Hence

$$M_{halo}(x) = M_{halo}(1)(1+a^2)\frac{x^3}{x^2+a^2} \tag{9}$$

(for a spherically symmetric halo) and

$$\rho = \frac{1}{4\pi R^2}\frac{dM}{dR} \propto \frac{x^2+3a^2}{(x^2+a^2)^2} \tag{10}$$

with very little in common with the NFW theoretical halos (see later). They are reasonable, because

- The x-derivative of $\rho(x)$ vanishes at $x = 0$, which is physically satisfactory.
- Then, for $x \ll a$ the density slowly decreases; for $x = a$, the density is still 1/3 of the central value, i.e. there is a "core" of radius a, which is therefore called the halo core radius.
- The density does not vanish for any value of x, i.e. there is no sharp boundary. The density always decreases.
- For $x \gg a$, the density decreases as x^{-2} (compared to x^{-3} in NFW halos).

This is reminiscent of the non-singular isothermal sphere, with a faster decrease from the centre out to the core radius, both of which for large x obey $\rho \propto x^{-2}$.

As $a \propto L^{1/5}$, i.e. low luminosity galaxies are much more concentrated. For a galaxy with L_* the core is of the order of the optical radius.

A non-physical property of the PSS halo density profile is that M does not converge for very large values of x, but rather linearly increases with x, with the mass of any halo being infinite. To surmount this difficulty the halo mass was defined as that at R_{200}, where R_{200} is the radius of a sphere within which the mean density is 200 times the mean density of the Universe, as also defined in theoretical models. Then

$$M_{halo} \approx M_{200} = M(R = R_{200}) = \frac{4}{3}\pi R_{200}^3 200\rho_c \tag{11}$$

where ρ_c is the critical density of the Universe. We see therefore that $M_{200} \propto R_{200}^3$. Following equation (5), V_{200} (the circular velocity at R_{200}) is a complicated function of R_{200}, but according to these authors, it can be approximated to

$$R_{200} = 250\left(\frac{L}{L_*}\right)^{0.2} \tag{12}$$

As the exponent, 0.2, is very small, the radii of the halos are relatively independent of the luminosity. A galaxy with $10L_*$ would have a halo only 1.5 times larger than a galaxy with luminosity L_* (this cannot be checked directly as also equation (5) cannot be applied to galaxies with $L > 4.33L_*$). The authors propose

$$M_{200} = 2 \times 10^{12} \left(\frac{L}{L_*}\right)^{0.5} M_\odot \tag{13}$$

(Note, however, that from $R_{200} \propto L^{0.2}$, together with the exact relation $M_{200} \propto R_{200}^3$, we should obtain $M_{200} \propto L^{0.6}$. The small difference in these exponents – 0.5 and 0.6– arises from the complexity of the problem). Therefore, the brighter galaxies have a halo that is more massive, but only slightly larger. The mass-luminosity ratio is then

$$\frac{M_{200}}{L} \approx 75 \left(\frac{L}{L_*}\right)^{-0.5} \tag{14}$$

Brighter galaxies have smaller mass-to-light ratios, hence the dark matter has more dominant effects in small or low-surface brightness galaxies. We can also calculate the luminous to dark matter ratio

$$\frac{M_{lum}}{M_{200}} = 0.05 \left(\frac{L}{L_*}\right)^{0.8} \tag{15}$$

Bright galaxies have relatively smaller dark matter halos, while the very bright galaxies nearly reach the maximum M_{lum}/M_{dark} ratio (~ 0.1) established from primordial nucleosynthesis models for the baryonic Ω_B.

For $x \gg a$, a constant value of V_{halo} is obtained $V_{halo}^2 = V^2(R_{opt})(1 - \beta)(1 + a^2)$. For galaxies with $L \sim L_*$, it is obtained that $V_{200} \sim V(R_{opt})$ which is readily interpreted: if a constant $V(R_{opt})$ is observed in a region already dominated by dark matter, it should be related to the halo circular velocity at large distances.

As $V_{200}^2 = GM_{200}/R_{200} \propto L^{0.5}/L^{0.2} = L^{0.3}$ we should have $V_{200} \propto L^{0.15}$. In binary galaxies, which are considered later, V_{200} could be identified with the orbital velocity of the secondary galaxy, statistically related to the difference of the two projected velocities along the line-of-sight. L would be the luminosity of the primary. A correlation between L and V_{200} has not been found (e.g. Zaritsky, 1997). This is, in part, justified as the exponent, 0.15, is so small that the orbital velocities are nearly independent of the luminosities. Theoretical models also agree in this respect.

Note that, if instead of $M_{200} \propto L^{0.5}$ as determined by these authors, we had taken $M_{200} \propto L^{0.6}$ from the definition of M_{200} as mentioned above, we would have obtained $V_{200} \propto L^{0.2}$, and $L \propto V_{200}^5$, closer to the observational infrared Tully-Fisher relation $L \propto V^4(R_{opt})$, if $V(R_{opt})$ were close to V_{200}.

White et al. (1983) and Ashman (1992) proposed $M_{200}/L \propto L^{-3/4}$, in which case $R_{200} \propto L^{0.08}$, indicating a lower dependence of R_{200} on L, and $V_{200} \propto L^{0.08}$. We will see later that these undetected correlations have a natural explanation in the magnetic model of the rotation curves.

The universal rotation curves also give two relations

$$\frac{R_{core}}{R_{200}} = 0.075 \left(\frac{M_{200}}{10^{12} M_\odot} \right)^{0.6} \tag{16}$$

$$\rho_{halo}(0) = 6.3 \times 10^4 \rho_c \left(\frac{M_{200}}{10^{12} M_\odot} \right)^{-1.3} \tag{17}$$

where ρ_c is the critical density of the Universe. Hence, brighter galaxies have relatively large core radii and small values for the central halo density. Therefore, even the central region of low-brightness galaxies is dominated by dark matter, while bright galaxies have their internal regions dominated by visible matter. These relations are important and confirmed by the NFW theoretical halos, even if the universal rotation curves do not have much in common with those deduced by the former.

The same formulae are valid for low-surface-brightness galaxies. In this case, the dark matter would be completely dominant, with a core radius of about 5-6 kpc.

Summarizing, the most interesting fact in the fitting effort made by these authors is that such a large variety of galactic types have rotation curves which can be adjusted to a single universal rotation curve (even for ellipticals, not considered here). This fitting assumes the existence of a dark halo that does not coincide with the universal halo profiles obtained by most theoretical models but which is very reasonable (except, perhaps, in that they have an infinite mass, which in practice is not a real problem). The explanation of the puzzling behaviour of binary systems, however, is still not completely satisfactory.

There is another general comment to be made. The universal rotation curve is a fitting problem. But this fitting should be interpreted in other models in a different way. Therefore, even if terms like "dark matter" and "dark halos" are used, this fitting does not prove the existence of dark matter in galaxies.

Bosma (1998) considered that the notion of universal rotation curves breaks down. He observed several galaxies with a high rotation velocity, but non-declining rotation curves. This could be due to the inclusion in Persic and Salucci's sample of very inclined galaxies, where opacity problems are difficult to handle when using H_α rotation curves. Verheijen (1997) also found 10 out of 30 galaxies in the Ursa Major clusters for which the rotation curves do not fit the universal rotation curves. Despite all these exceptions the scheme introduced by Persic, Salucci and collaborators, provides a first fit that theoretical models should take into account.

2.5 Dwarf irregular galaxies

Dwarf irregular galaxies can be considered extreme late-type spirals, at least as concerns the rotation curve and the associated dark matter. Many of them present well defined rotation curves that can be obtained from HI measurements.

With respect to normal spirals, dwarf irregulars have the advantage of presenting a large gaseous component, and so rotation curves can be traced better and to much larger radii, up to 17 radial scale lengths. Indeed, the rim of the halo has probably been detected (Ashman, 1992).

Kerr, Hindman and Robinson (1954) and Kerr and de Vaucouleurs (1955) showed that the LMC and the SMC were rotating. This result was extended to other irregulars showing that most late-type dwarf galaxies rotate, although their rotation velocities are lower, of the order of $\sim 60kms^{-1}$. It was also established that the rotation curve rose slowly to the last measured point. These galaxies were soon considered ideal to study galactic dark matter, not only because the absence of a bulge made analysis simpler, but mainly because the rising curves greatly differed from the expected Keplerian decline. First results (Carignan, 1985; Carignan, Sancisi and van Albada, 1988) seemed to indicate that these galaxies have dark matter properties similar to those of normal spirals and that the inner parts do not require great amounts of dark matter. This trend was not confirmed later, and the commonly accepted picture was that the contribution of the disk is insignificant and that they are dominated by dark matter at all radii. For more details of this history see the thesis by Swaters (1999).

But this conclusion was based on a very small sample of galaxies. The studies by Broeils (1992) and Coté (1995) were based on only eight late-type dwarf galaxies, a small number taking into account the large spread of dark matter properties that this type of galaxies presents. Recently, Swaters (1999) has carried out the greatest effort made to date to systematically observe and analyze this problem. It is also important to have a large sample observed and reduced with the same techniques. To determine the dark matter amounts, it is necessary to obtain photometric maps. In this study, this was done for 171 galaxies at the 2.5m INT at La Palma. Of these, 73 were observed in HI with the Westerbrok Synthesis Radio Telescope. Rotation curves were obtained for 60 of them, and detailed dark matter models were carried out for 35. Clearly, the results obtained in this work are based on the largest and most homogeneous sample. In general, these results did not confirm the previous widely accepted picture; late-type dwarfs are not essentially different from normal brighter spirals, which is more in agreement with the pioneering interpretations.

Despite their apparent loss of symmetry, the exponential decline of typical disks is usually observed in these HI rich galaxies. Swaters found that the rotation curves flatten after about two disk scale lengths. There are several galaxies with fairly flat rotation curves with amplitudes as low as 60 kms^{-1}. The main difference with rotation curves of spiral galaxies is that no cases of declining curves were found, which was explained by the fact that these galaxies have no bulge at all or only a small one while bright spirals with declining curves do have a large bulge.

The outer slope as a function of R-magnitude is plotted in figure 9 and includes both bright galaxies (from Broeils, 1992) and galaxies belonging to the

Ursa Major cluster (Verheijen, 1997). It is seen that the variation in slopes is larger in late-type spirals.

In Fig. 10 we reproduce the Tully-Fisher relation from Swaters (1999) that extends the relation to fainter types. We observe $L \propto V_{max}^{\alpha}$, where $\alpha \sim 4.4$. It is also observed that late-type dwarfs rotate noticeably faster than predicted by the Tully-Fisher relation.

In general, late-type galaxies are not dominated by dark matter within the optical disk for radii less than about four scale lengths. The required stellar mass-to-light ratio is however greater than in bright spirals, of the order of 10, reaching values as high as 15. Maximum disk models fit the obtained rotation curves reasonably well, but other models cannot be excluded.

Many irregulars are satellites of bright galaxies or at least of a small group of galaxies as in the Local Group. Here, a gradation in the properties from dEs to dIrrs would support the hypothesis that Irrs could eventually evolve into dEs (e.g. Aparicio et al. 1997; Martinez-Delgado 1999). Phoenix could be a clear example of an intermediate type. Moreover, dEs are preferentially distributed close to the largest galaxies, while dIrrs are found in the outskirts (the Magellanic Clouds are exceptions) (van dem Bergh 1999).

Salucci and Persic (1997) proposed that the "universal rotation curve" was also valid for dwarf irregulars, though then the large amount of data in the thesis of Swaters was not available. In this case, the calculation of a and β is given by different formulae:

$$a = 0.93 \left(\frac{V_{opt}}{63kms^{-1}} \right)^{-0.5} \tag{18}$$

$$\beta = 0.08 \left(\frac{V_{opt}}{63kms^{-1}} \right)^{1.2} \tag{19}$$

if

$$V_{opt} = 63 \left(\frac{L}{0.04L_*} \right)^{0.16} kms^{-1} \tag{20}$$

Thus, for a bright dwarf irregular, $L \sim 0.04L_*$, $V_{opt} \sim 63kms^{-1}$, characteristic values are $a = 0.93$ and β=0.08. Therefore the core radius is nearly as large as the optical radius and the contribution of the visible matter at the optical radius is nearly negligible. If $L < 0.04L_*$, β is even lower and a higher. Under this interpretation, therefore, dwarf irregulars are very dark galaxies, have very dense halos and large masses, obtainable with $8 \times 10^{10}(L/0.04L_*)^{1/3}$.

Salucci and Persic (1997) give a formula to estimate the total mass of a galaxy as a function of its visible mass

$$M_{200} = 3 \times 10^{12} \left(\frac{M_{visible}}{2 \times 10^{11} M_{\odot}} \right)^{0.4} M_{\odot} \tag{21}$$

Thus, when $M_{visible}$ is large the $M_{200}/M_{visible}$ ratio decreases. Merely to state that M_{total} is proportional to L, as is often done, would be to make a bad assumption, worse than supposing that all galaxies have equal mass, irrespective of their luminosity.

Though most studies of these galaxies conclude that moderate or large amounts of dark matter are required, we cannot exclude the magnetic interpretation of the data for spiral galaxies, which should also be taken into account for dwarf irregulars. Under this interpretation, the higher magnetic fields required imply larger escaping fluxes that are actually observed, for instance in M82 (also associated with the ejection of magnetic fields (Reuter et al., 1982; Kronberg and Lesch, 1997) or in NGC 1705 (Meurer, Staveley-Smith and Killeen, 1998), a galaxy requiring specially high DM central density (0.1 $M_{\odot} pc^{-3}$) and a large mass loss rate of the order of 0.2-2 $M_{\odot} yr^{-1}$.

2.6 The rotation curve of the Milky Way

Paradoxically, the rotation curve of the nearest galaxy remains poorly known. Extinction is too large to observe the stars and too small to observe the gas. It is preferable to observe the gas, either at 21 cm or at 2.7 mm, because it extends at much greater radii. Thus we must rely on the corotation of both the stellar and the gaseous systems, an assumption that is not always justified, as mentioned previously. The tangent-point method to obtain the rotation curve for $R < R_0$, with R_0 being the solar galactocentric distance, is well known and need not be repeated here in detail.

The points of the circle with a Galactic Centre-Sun diameter are characterized by a radial velocity from the Sun equal to their rotation velocity, and this velocity is determined by the fact that it corresponds to the largest redshift (in the first quadrant) or the largest blueshift (in the fourth quadrant). The different values at the points of this circle give us the rotation curve.

However, this method only provides the rotation curve out to 8 kpc, but to analyze our dark halo and the mass of the Milky Way itself, this is too small. To extend the rotation curve to larger galactocentric distances, a variety of objects have been observed. These objects have to be bright, to be observed from afar, their distance must be accurately determined (which in practice is the largest source of error) and their radial velocities must be easily obtainable. Carbon stars, OB stars, planetary nebulae, cepheids and HII regions have been used to study the outer Galaxy, but the errors are large and the maximum distance is less than $2R_0$. A complete account of these attempts was given in the review by Fich and Tremaine (1991) and will not be repeated here. There is a crucial date (1965) prior to which, as reviewed by Schmidt (1965), it was thought that the outer rotation curve was Keplerian and the estimated mass of the Milky Way was about $2 \times 10^{11} M_{\odot}$. After this year, various authors began to realize that the outer curve was more or less flat, and the conclusion that our Milky Way may contain large amounts of dark matter became more and more widely accepted.

The best method to investigate the outer rotation curve was proposed by Merrifield (1992), who considered that a ring in the Milky Way with constant vertical scale length, h_z, has a variable angular size when seen from the Sun; or in his own words "much as a person standing in a volcano might estimate his or her location within the crater from the variations in the apparent height of the walls in different directions". It is easily obtained that

$$\frac{v_r}{\sin l \cos b} = \frac{R_0}{R}\theta(R) - \theta_0 \equiv W(R) \tag{22}$$

where v_r is the radial velocity from the Sun, $\theta(R)$ is the rotation at a point with galactocentric distance R, θ_0 is the rotation velocity at $R = R_0$ and l and b are the galactic longitude and latitude. Therefore, if we have a data cube $T_b(v_r, l, b)$, where T_b is the HI brightness temperature, it is possible to divide the cube into slices with constant $W(R)$ as defined in the above equation. $W(R)$ only depends on R and we may use h_z to know exactly what value of R we are speaking about. From our location in the Galaxy, the HI layer of thickness h_z at some point of radius R will present an angular size in galactic latitude of

$$h_b = 2\tan^{-1}\left(\frac{h_z/2R_0}{\cos l + [(R/R_0)^2 - \sin^2 l]^{1/2}}\right) \tag{23}$$

If we then take a constant-W slice, obtain the variation in angular width as a function of longitude l, and compare it by this formula, we can calculate it by fitting the value of R/R_0 (and even h_z/R_0) of the slice, and hence obtain $v_r(R/R_0)$ and $h_z(R/R_0)$.

There are some inherent problems. The orbits must be circular and vertical shear must be absent, i.e. v_r should not depend on z. The galactic warp introduces further complications, although these can be overcome. In such a way, Merrifield was able to reach points in the Milky Way rotation curve out to about 20 kpc, or 2.5 R_0, with an unprecedented degree of precision.

The results greatly depend on the values of R_0 and θ_0. Merrifield proposed $R_0 = 7.9 \pm 0.8 kpc$ and $\theta_0 = 200 \pm 10 kms^{-1}$, rather lower than usually recommended in other works, to match other kinematic constraints and in line with the rotation curves of similar galaxies. More recently, Honma and Sofue (1996, 1997) have used Merrifield's method to estimate the rotation curve, the geometry of the halo and the total mass of the Milky Way, investigating their uncertainties. Errors in R_0 are relatively unimportant because they just change the scaling in the radial direction, but changes in θ_0 produce highly different interpretations of our hypothetical halo. In Fig. 11 we plot their results for three different values of θ_0: 220, 200 and 180 km s^{-1}.

The rotation velocity decreases beyond $2R_0$ for all cases. Reasonable Keplerian fits are obtained for $R \geq 2R_0$ if θ_0 is in the range 200-207 kms^{-1}. If $\theta_0 < 200 kms^{-1}$ the curve declines faster than Keplerian.

Using $\theta_0 = 220kms^{-1}$, as recommended by the IAU, the outer rotation curve rises between $R = 1.1R_0$ and $2R_0$, which is uncommon in other galaxies of the same type, having a flat curve within the optical disk. To obtain a flat rotation curve, θ_0 should be as small as 192 km/s.

R_0 and θ_0 are related to Oort's constants A and B (note $A - B = \theta_0/R_0$), which are fairly well determined. (θ_0/R_0 could also be directly determined by means of the VLBI determination of the proper motion of Sgr B2, taking two quasars behind as reference. Accurate data, in this respect, will be available in the near future. Honma and Sofue (1996) propose $\theta_0 = 200km$ and therefore $R_0 = 7.6kpc$, based on this result and those obtained by other authors also claiming lower θ_0 and R_0.

Assuming a spherical mass distribution they obtain for the mass of the Milky Way a low value of $2.0 \pm 0.3 \times 10^{11} M_\odot$, which is close to the early estimates.

The Keplerian rotation curve does not require dark matter beyond $2R_0$, but it would still be necessary within $2R_0$, because an exponential disk has a rotation curve declining beyond 2.2r, when r is the radial scale length, in conflict with the flat rotation out to $2R_0$. However, the dark matter needed could be much less than previously calculated. On the other hand, the shape of the dark halo would differ greatly from that theoretically deduced.

3 Dark matter in other galaxies

Although this paper considers spiral galaxies and rotation curves, this type of galaxies may have a lot in common with other types, and therefore these must be commented as well, in particular from the point of view of their dark matter content. Moreover, observations other than rotation curves have inspired a long list of possible methods to test for dark matter.

3.1 Dark matter in elliptical galaxies

There are basically three methods for specifically estimating the mass of an elliptical galaxy:

a) From the stellar velocity dispersion.

b) From the neutral gas velocities found in the outermost region, in certain galaxies.

c) From the X-ray corona surrounding all ellipticals.

There also exist complementary methods, using observations of ionized gas in the central parts, globular clusters, gravitational lensing, theoretical considerations about the bar instability and the chemical evolution.

The general conclusion, taking into account all these studies, could be, in summary, that dark matter amounts comparable to visible matter could be present in the visible part of the galaxy, and that larger dark matter amounts, probably as large as in spirals, are present in a halo surrounding the galaxy, but that, in any case, the evidence of dark matter in ellipticals is less than in the case of spirals. Even the complete absence of dark matter cannot be easily ruled out.

Several reviews have been written on the topic (e.g. Ashman, 1992; Trimble, 1987; de Zeeuw, 1992; Kent, 1990; Bertin and Stavielli, 1993). Let us remember that the surface brightness of an elliptical galaxy can be fitted by de Vaucouleurs' law

$$I(R) = I_e e^{-7.67\left((R/R_e)^{1/4}-1\right)} \tag{24}$$

(de Vaucouleurs, 1948), where R_e is the radius enclosing half of the light and $I_e = I(R = R_e)$ is another constant. The value of R_e is often used as a parameter that normalizes all lengths as does the radial scale length in spirals. This law seems to be rather well matched, but it is just one fitting which might be less appropriate for some subtypes (Andreakis, Peletier and Balcells, 1995).

Let us comment on the three basic methods, and more briefly about other methods:

a) The observations of stellar velocity dispersion, interpreted in terms of Jeans' equation or of the Virial theorem, can provide the total mass for $R < R_e$, or even at larger distances.

The Virial theorem for a spherical, steady-state, static isothermal elliptical galaxy reduces to the simple expression

$$2R \approx \frac{GM}{\sigma^2} \tag{25}$$

where R is an equivalent radius. For a given R, $\sigma^2 \propto M$, because the stellar chaotic thermal velocities, quantified by the velocity dispersion, σ, must prevent gravitational collapse. The larger the mass, the larger the stellar velocities must be. This formula gives a first approximate mass. In practice however, much more sophisticated models than this one are used to interpret the velocity dispersions. There is a "degeneracy" between the unknown anisotropy and the unknown gravitational potential. If the anisotropy of the orbits is known the potential can be determined, but not both simultaneously. We should know if orbits are mainly circular, or mainly radial, or something in between. The anisotropy is characterized by the parameter β, which is defined later, in Section 3.5.2.

Pioneering works by Binney, Davies and Illingworth (1990), van der Marel, Binney and Davies (1990) and others have concluded that no gradients in M/L were clearly appreciated and that no dark matter was needed to explain the central surface brightness and the velocity dispersions. The M/L values are of the order of 12h (Binney and Tremaine, 1987) (about 8 for h=0.65) which is comparable to the solar neighbourhood values. It is slightly higher, but this fact can be explained mainly by the absence of young stars in ellipticals. One-component models, without any halo, provide a good zeroth-order description (Bertin, Saglia and Stiavelli, 1992).

Bertin, Saglia and Stiavelli (1992) also considered two-component spherically symmetric collisionless self-consistent models, which were later used to interpret real data from 10 bright selected galaxies (Saglia, Bertin and Stiavelli, 1992) and found some evidence for dark matter to be of the order of the visible mass. The presence of rotation and of tangential anisotropy makes it difficult to draw firm conclusions.

As in the case of spirals with their rotation curve, a flat or slowly increasing velocity dispersion, $\sigma(r)$, may indicate dark halos dominating the dynamics (Saglia et al. 1993) but there is a surprisingly large variety of σ-profiles, some of which decrease outwards relatively fast. Therefore, Saglia et al. did not find any compelling evidence of dark matter out to 1-2 R_e. Carollo et al. (1995) observed flat or gently declining velocity dispersion profiles in four elliptical galaxies, concluding that massive dark halos must be present in three of the four galaxies, although no clear conclusion was obtained for the fourth. Bertin et al. (1994) found that in a sample of 6 galaxies, three of them were not suitable for reliable modelling, two of them presented no evidence for dark matter and one (NGC 7796) appeared to have a distinct dark halo. But the conclusion that some galaxies have a dark halo while others do not is problematic for understanding what an elliptical galaxy is. De Paolis, Ingrosso and Strafella (1995) found that dark matter inside R_e is negligible with respect to the visible mass.

b) A small fraction of elliptical galaxies are surrounded by a ring of neutral hydrogen, for instance, NGC 1052, NGC 4278 and NGC 5128. In these cases, the determination of a dark matter halo is very similar to its determination in spiral galaxies, from the rotation curve. One of the best studied gaseous rings is that of IC 2006 (Schweizer, van Gorkom and Seitzer, 1989). The neutral gas counter-rotates at a radius of 18.9 kpc (6.5 R_e) and has a total mass of $4.8 \times 10^8 M_\odot$. This galaxy also has a counter-rotating central mass of ionized gas out to $\sim$5 kpc. These gaseous components of some ellipticals have either been accreted or are the remnant of a merger from which the elliptical was created.

Schweizer, van Gorkom and Seitzer (1989) found evidence for a DM halo in IC 2006 with twice the mass of the luminous matter within 6.5 R_e, under the assumption that the HI ring is flat and circular. Bertola et al. (1993) analyzed five elliptical galaxies, combining the M/L ratios obtained with the inner ionized hydrogen component and the outer neutral hydrogen ring. M/L is constant out to about R_e with a moderate value of 3.5 ± 0.9 but becomes very large in the ring region. These authors found a similarity in the distribution of dark matter in ellipticals and in spirals, suggesting a similar picture for the origin of both.

As we will discuss later, magnetic fields may explain rotation curves without requiring dark matter in spirals. Similar arguments can be considered to interpret gaseous rings around ellipticals. In particular, a narrow ring is pushed towards the centre more easily than a disk, because the outward magnetic pressure force need not be compensated by a magnetic tension. It is to be emphasized that the IC 2006 gaseous ring is very narrow, and is not even resolved by VLA.

c) The most promising method to study dark matter in ellipticals is based on the existence of X-ray halos. A hot X-ray emitting gas typically extends out to 50 kpc (Forman, Jones and Tucker 1985). The probable origin of the gas is mass loss from stars; supernovae heat it up to $\sim 10^7 K$, bremsstrahlung being the main cooling process (Binney and Tremaine, 1987). Typical masses of this hot gas are $10^{10} M_\odot$.

Hydrostatic equilibrium is usually assumed for the gas. Then, for a spherical DM halo

$$M(R) = -\frac{kTR}{Gm}\left[\frac{d\ln\rho}{d\ln R} + \frac{d\ln T}{d\ln R}\right] \tag{26}$$

where ρ is the density of the gas. Once $M(R)$ is determined in this way, we obtain the DM halo profile.

The gas is not in perfect hydrostatic equilibrium. The innermost gas in the X-ray halo is more efficiently cooled, because cooling is proportional to the electron density, which is still high. An inwards flow in the inner region is therefore to be expected (Binney and Tremaine, 1987). Cooling flows have been observed (Mushotzky et al. 1994) and models including radial flows have been developed (e.g. Ciotti et al. 1991). The equilibrium probably breaks down in galaxies with low X-ray-to-optical luminosity ratios. Nevertheless, hydrostatic equilibrium is generally assumed.

In the above formula, the temperature profile $T(R)$ is not provided by the observations with enough precision. The strengths of some X-ray lines or the shape of the X-ray continuum should provide this T-profile but, in practice, this is still rather problematic. For giant cD galaxies, like M87, the temperature is exceptionally well determined and the method provides more reliable results. For M87 the data are spectacular: $M(R < 300kpc) \sim 3 \times 10^{13} M_\odot$; the mass-to-light ratio reaches a value of 750; about 95% of M87 mass is dark matter (Fabricant and Gorenstein, 1983; Stewart et al. 1984; Binney and Cowie, 1981). However, cD galaxies may be exceptional; as they lie at the centre of a rich cluster, the DM encountered could belong to the cluster as a whole. Below, we address this problem in Section 5.

Difficulties arise in the analysis of normal ellipticals. If $T(r)$ is unknown, it is tempting to assume an isothermal distribution (e.g. Forman, Jones and Tucker, 1985), which might be justifiable. Mushotzky et al. (1994) were able to obtain 6 points of $T(R)$ in NGC 4636, finding that T was approximately constant. Moreover Matsushita (1997) and Jones and Forman (1994) confirmed the constancy of $T(R)$. High M/L ratios are in general obtained, in the range 10-80, especially at large distances, but Trinchieri, Fabbiano and Canizares (1986) concluded that DM halos were not absolutely required by the data. Fabbiano (1989) also found much lower masses.

Furthermore, the contribution of unresolved discrete X-ray sources, such as accreting binaries, complicates the analysis (de Paolis, Ingrosso and Strafella, 1995), which could be related to the fact that the relative amount of DM is higher for X-ray bright ellipticals.

Models often take as a boundary condition that X-ray emitting gas is confined by the cluster intergalactic gaseous pressure (Fabian et al. 1986). Other authors assume a vanishing pressure at infinity (Loewenstein and White, 1999).

The gas responsible for the X-ray emission cannot rotate very fast and hence no dynamo can generate magnetic fields capable of affecting the hydrostatic equilibrium. However, intergalactic magnetic fields could have an influence as a boundary condition. For the intracluster intergalactic space, with $n \sim 10^{-5} cm^{-3}$ and $T \sim 10^7 K$ the thermal pressure is of the order of $10^{-14} dyncm^{-2}$. As discussed below, cluster intergalactic fields are of the order of 10^{-6} G, and therefore the magnetic energy density is of the order of the thermal pressure. External magnetic fields could contribute to confining the X-ray emanating hot gas, thus reducing the large amounts of dark matter required. This external field would not act isotropically and would produce eccentric X-ray isophotes, such as for instance in NGC 720. Eilek (1999) makes suggestions about the importance of magnetic fields in the dynamics of clusters which are relevant to the dynamics of X-ray halos around giant ellipticals at the centre of clusters, where the field can provide an important part of the pressure support.

Buote and Canizares (1994) observed a different isophote geometry for X-rays and for the optical in NGC 720. The X-ray isophotes are more elongated and their major axes are misaligned by about 30^o. If the total matter were distributed as is the optical light, it could not produce the observed ellipticities of the X-ray isophotes. They interpreted this ellipticity as being produced by a dark matter halo and developed a model that did not need the $T(R)$ profile, and which also favoured the existence of a large dark matter halo. Davis and White (1996) and Loewenstein and White (1999), too, developed methods not requiring the temperature profile that imply DM halos.

d) In addition to these basic methods there are others that should be mentioned. The image splitting of an individual gravitational lens system consisting of an elliptical is only slightly sensitive to the existence of a DM halo, and so, one cannot definitely discriminate between galaxies with and without halos, with some exceptions (Breimer and Sanders, 1993; Kochanek, 1995). Indeed, in three cases where the lens is clearly a single galaxy, there is no need to consider any dark matter halo. Maoz and Rix (1993), however, deduce from gravitational lensing methods that $M(R)$ increases linearly with R, as is typical in isothermal halos.

Globular clusters have been considered to deduce the existence of dark matter halos in ellipticals, mainly in M87 (Huchra and Brodie, 1987; Mould et al., 1990). They support the conclusions obtained by other methods: models without dark halos do not fit the data in M87, but they cannot be excluded in NBC 4472 (Mould et al. 1990). This problem is considered in Section 2.6. Planetary nebulae have also been considered in NBC 5128 by Ford et al. (1989)

and by others, who found a radial increase in (M/L_B) reaching values of about 10, although de Zeeuw (1992) suggested a lower gradient. Ciardullo and Jacoby (1993) deduced that the non-interacting elliptical galaxy NGC 3379 has no dark matter halo, and that a constant M/L of about 7 explained the observations perfectly. Theoretical studies of bar instability (Stiavelli and Sparke 1991) and chemical evolution (Matteuci 1992) were unable to unambiguously determine the presence of a dark halo.

In conclusion, elliptical galaxies could have dark matter halos similar in mass and extent to those in spiral galaxies (Danziger, 1997), but the evidence is not so clear and it cannot even be completely rejected that they possess no dark halo at all. As exceptions, in giant cD galaxies like M87, the existence of large amounts of dark matter seems to be fully demonstrated.

3.2 Lenticular Galaxies

Like ellipticals, SO galaxies present problems in the detection of a dark matter halo. Like ellipticals, SO galaxies were considered to possess low amounts of gas, but new techniques are able to observe sufficient quantities of gas to determine the rotation curve and hence dark matter, under the standard interpretation for spirals. The general conclusion is that these galaxies also have dark matter halos, but that they may be relatively smaller, as these galaxies are bright.

Estimations of DM in SO galaxies are more promising in these exceptional lenticulars that have a large amount of gas. The gas is distributed in a large outer ring, at about twice the optical radius, often warped with respect to the optical plane, and is possibly of external origin. Van Driel and van Woerden (1991, 1997) have studied gas-rich lenticulars. NGC 2787, 4262 and 5084 have large M/L ratios at twice the optical radius (about 25-30). A problem encountered in this study was that no surface photometry was available to carry out standard analysis, except for NGC 4203. The large DM halos needed were surprising for these gas-rich lenticulars, compared with normal lenticulars. Magnetic fields could again introduce a different interpretation as gaseous rings are subject to magnetic centripetal forces.

NGC 404 has been measured in HI by del Rio et al. (1999), who found a large M_{HI}/L_B ratio of the order of 0.2, mainly contained in two rings. The most important fact under our present perspective is that this galaxy declines with a near perfect Keplerian profile, the Keplerian fit being characterized by a correlation coefficient of 0.9 (see Fig. 12 which was interpreted by these authors as due to a central mass concentration with no dark matter).

Pignatelli and Galletta (1997) obtained dynamical models of SO and Sa galaxies that do not need a dark matter halo.

3.3 Dwarf spheroidal galaxies

This type of galaxy is probably the most common in the Universe. Despite their low luminosity, they may contain large amounts of dark matter, and thus contribute greatly to the mass of the Universe. However, dwarf spheroidals do not possess gas in the periphery, as do bright ellipticals. Therefore, the determination of DM is more problematic. There are basically two methods for detecting dark matter in this type of galaxies:

a) Tidal radii.- The dwarf spheroidal satellites of the Milky Way, for instance, could become tidally disrupted if they did not have enough dark matter, thus increasing autogravitation and preventing it. Let us determine the radius of the satellite necessary for autogravitation to match tidal disruption, by means of a rough model.

Suppose a satellite dwarf with mass m and radius r orbiting around the primary galaxy with mass M, with R being the distance between the two galaxies. Suppose the dwarf divided into two halves. They would attract one another with a force of the order of Gm^2/r^2. The tidal disrupting force would be the difference in gravitational force produced by the primary $\frac{GMm}{2}\left(\frac{1}{(R-r)^2} - \frac{1}{(R+r)^2}\right) \approx GMmr/r^3$. The two forces become equal when

$$r = R\left(\frac{m}{M}\right)^{1/3} \tag{27}$$

More precise calculations (e.g. Binney and Tremaine, 1987) give the same orders of magnitude. As the dwarf is not a rigid body, at a galactocentric radius equal to this critical value, r, stars would escape and would be trapped in the gravitational field of the primary. Therefore, at a “tidal radius”, r, the density should drop to zero. From the observational point of view, the tidal radius is difficult to determine, and so it must be obtained by extrapolation. It seems to be too large, when m is obtained from the surface brightness with a constant M/L relation.

Hodge and Michie (1969) first detected a greater than expected tidal radius for Ursa Minor, and Faber and Lin (1983) first used this procedure for estimating dark matter. More (1996) used the observation of stars being removed in the tidal tails of dwarf galaxies to conclude that their dark matter halos must be truncated at 400 pc, limiting their M/L ratio to less than about 100. Burkert (1997) investigated four dwarf spheroidal galaxies orbiting around the Milky Way: Sextans, Carina, Ursa Minor and Draco, considering their tidal radii, and concluded that Sextans is dark matter dominated but not the other three.

It should be emphasized that the mass obtained depends on the third power of the estimated tidal radius, which is an important source of errors. On the other hand, what is actually estimated is the m/M ratio. If the Milky Way mass were overestimated, so would be the mass of the satellite. If the Milky Way contained no dark matter the satellite in turn would not require this component.

b) The velocity dispersion of the stellar system.- This method has much in common with that used for the central parts of other galaxies. But in others, the analysis is complemented with peripherical effects which are now completely absent. The study of the dark matter in dwarf spheroidals basically rests on the assumption that its distribution and that of the stars are similar, a condition which is probably unrealistic, unless this type of galaxy is the only exception. Nevertheless, most workers on this topic generally agree that these galaxies have a large DM content, maybe 10 times higher than luminous matter; indeed, ratios of 100 have been reported as well as very high central condensations in the range $(0.1 - 1M_{\odot}pc^{-3})$ (Mateo et al. 1992).

The possibility that all dwarf galaxies may have the same mass, despite their large luminosity range, has been proposed (see Ashman, 1983). Mateo (1997) obtains a mass of $2 \times 10^7 M_{\odot}$ for all dwarf spheroidals irrespective of their luminosity. Salucci and Persic (1997) obtain $M \propto L^{1/4}$. Among the observational difficulties we should also mention the fact that velocity dispersions are low, of the order of 10 kms^{-1}, and therefore, a high spectral resolution is required.

If luminous and dark matter have the same spatial distribution, it is easy to deduce the central density and the central mass-to-luminosity ratio. From the Virial theorem, we straightforwardly deduce

$$\rho_0 = \frac{9\sigma_0^2}{4\pi G R_c^2} \tag{28}$$

where ρ_0 is the central density, σ_0 the central velocity dispersion and R_c an equivalent radius, or core radius, identifiable with the radius at which the surface brightness is one half the central value. Within this radius, the mass would be $\rho_0 \frac{4}{3}\pi R_c^3$ and the luminosity, $\Sigma_0 \pi R_c^2$, where Σ_0 would be the observable central surface brightness. Hence, the M/L fraction would be

$$\frac{M}{L} = \frac{3\sigma_0^2}{\pi G \Sigma_0 r_c} \tag{29}$$

The constant should be $9/2\pi$, instead $3/\pi$, as deduced by more detailed calculations (Richstone and Tremaine, 1986; Ashman, 1993).

An important study was made by Kuhn and Miller (1989), in which dwarf spheroidal galaxies were not considered as virialized systems. These galaxies could be unbound and losing stars. Numerical simulations have been carried out by Kroupa (1997) and Klesen and Kroupa (1998) in which the dwarf satellites are partially disrupted in perigalactic passages; orbiting condensations can be identified after this event near the satellite. Then, they might contain no dark matter at all and yet present a high stellar velocity dispersion. The tidal disruption of a satellite produces a remnant that contains about 1% of the initial mass.

Summarizing, dwarf spheroidal galaxies could contain large amounts and concentrations of dark matter, but severe observational difficulties prevent their precise determination. Even models with no dark matter at all cannot be excluded.

3.4 Polar ring galaxies

Some galaxies possess a polar ring, most of which are of type SO. The most widely accepted explanation for the formation of polar ring galaxies is that accreted gas settles onto orbits that are more frequently contained either within the equatorial plane or in polar planes. As Polar Ring Galaxies are typically SO galaxies, and the ring is gaseous, blue and undergoing star formation, their nature does not greatly differ from gas-rich SO galaxies, commented above, but the importance of Polar Ring Galaxies with respect to the problem of dark matter halos is that information about the overall gravitational potential can be obtained in two planes: the disk and the ring planes, thus potentially constraining the shape of the halo.

Sackett (1999) has reviewed the topic of the shape of halos, with Polar Ring Galaxies being one of the most interesting techniques to determine it. Assuming it to be a triaxial ellipsoid, the ovalness (b/a) and the flattening (c/a), where $a \geq b \geq c$, are to be determined. Intrinsic ovalness of the density distribution in the disk can be used to trace the non-axisymmetry of the halo in any face-on spiral galaxy (Rix and Zaritsky, 1995) finding a value for $b/a \sim 0.85$. This figure together with those obtained by other methods summarized by Sackett (1999) indicates that $b/a > 0.7$, so that the unobserved halo could have a higher axisymmetry.

However, the flatness is more difficult to assess, and polar ring galaxies are specially suitable for this purpose. This point is particularly important because it can provide information about the baryonic and dissipative nature of the halo dark matter (Pfenniger, Combes and Martinet, 1994).

Polar rings have very large radii, of about 20 stellar disk radial scale lengths, and therefore the perturbing influences of central luminous components are less important, and observations would provide the flatness c/a of the dark halo. This task has been carried out by several authors since the pioneering work by Schweizer, Whitmore and Rubin (1983) including more recent analyses by Combes and Arnaboldi (1996) and others (see the review by Sackett, 1999, and references therein).

From the analysis of polar ring galaxies, Sackett concludes that halos are highly flattened, $0.3 \leq c/a \leq 0.6$, which coincides with a similar conclusion from the flattening of the X-ray halos of elliptical galaxies (Buote and Canizares, 1998). Observations of gravitational lensing (Kochanek, 1995) also suggest greatly flattened halos.

Other methods to determine c/a not based on PRG have been reported. The conclusions are model dependent and, in some cases, are even based on hypotheses that are not completely demonstrated. For instance, the analysis of warps (New at al, 1998) and of flaring (Olling and Merrifield, 1997; Becquaert and Combes, 1998) are based on interesting models, but which are not free of alternative explanations.

Ashman (1992) points out that polar ring galaxies are unusual objects and therefore their hypothetical halos may be atypical. For instance, the merging process from which they have originated could have given rise to a flattened halo. Alternatively, the settling of a polar ring in the accretion process may require a flattened halo, in which case, the scarcity of polar ring galaxies would suggest that most halos are spherical. The influence of magnetic fields on the dynamics of these rings may be non ignorable.

As the polar ring contains HI, it is useful to detect dark matter, but as the host galaxy is lenticular and usually gas-poor, we cannot benefit from the standard analysis of spirals. Therefore the study of the exceptional galaxy NGC 660, the only polar-ring spiral galaxy known is very important. It has been extensively studied by van Driel et al. (1995) and van Driel and Combes (1997). The disk has a flat rotation curve and the polar ring is a rising one, which is rather puzzling. No conclusion about the flatness was reached, although the authors noted that several problems cannot be ignored, such as the fact that the ring is very massive, so that it cannot be considered to be formed by test particles tracing the potential, together with the fact that, obviously, the polar ring velocity and the disk velocity cannot be measured at the same radius. These objections raised by van Driel and Combes (1997) also hold for other dark matter studies in polar ring galaxies.

3.5 Binary galaxies

Binary stars constitute the best direct method to determine stellar masses. It is therefore to be expected that binary galaxies should provide galactic masses. The first and closest example is the double system formed by M31 and the Milky Way, and we should begin with this one.

3.5.1 M31 and the Milky Way

The Local Group contains more than 35 galaxies, most of which are dwarf ellipticals and irregulars with low mass; this complicated system may be considered as being formed by two main galaxies, M31 and the Milky Way, with other dynamically less important satellites belonging either to one of them or to the pair.

This picture is derived from galactic luminosities, but when possible dark matter is taken into account, it is not clear at all. M31 has a visible mass of about $4 \times 10^{11} M_\odot$. The Milky Way, $10^{11} M_\odot$. Next are M33 with $4 \times 10^{10} M_\odot$, LMC with about $2.3 \times 10^{10} M_\odot$, SMC with $6.3 \times 10^{9} M_\odot$, IC10 with $3 \times 10^{9} M_\odot$ and other minor members. Note that this list, when ordered following the total mass, could be changed. For instance, LMC has a visible mass of $\sim 1/5$ the mass of the Milky Way. As it has been suggested that irregulars may contain more dark matter than bright galaxies, the total mass of LMC could be as large as, or even more massive than, that of the Milky Way. In this case it could no longer be considered our "satellite". Let us however retain the more standard viewpoint and consider that M31 and the Milky Way are dynamically dominant and form a binary system.

M31 has a line-of sight velocity of $\sim$ -300 km s^{-1}, and therefore it is approaching us. Taking into account our motion of rotation within the galaxy of about $220 kms^{-1}$, it is easy to deduce that the speed of M31 with respect to the centre of our Galaxy is about $-125 kms^{-1}$. Both galaxies are approaching one another, with M31 therefore being an exception in the general motion of expansion of the Universe. There are different interpretations of this fact:

a) "Ships passing in the night"

Besides the expansion velocity following Hubble's law, galaxies have a peculiar velocity. For instance, our galaxy is moving with respect to the CMB black body at about $620 kms^{-1}$. Within a cluster, peculiar motions are also of the order of $600 kms^{-1}$. Even if these high velocities could be interpreted in other ways, such as bulk motions of large inhomogeneities or only characteristic of rich clusters, it is evident that some thermal-like peculiar velocities of this order of magnitude characterize the velocity dispersion of present galaxies, once the Hubble flow is subtracted. If we write for the velocity of a galaxy

$$v_i = H_0 r_i + V_i \tag{30}$$

where V_i is independent of r_i, for distances less than V/H_0, Hubble's law becomes imprecise and of little use, peculiar velocities being larger than expansion velocities. The law is imprecise for distances shorter than about 10 Mpc and becomes absolutely unsuitable for r<1 Mpc. Therefore, a simple interpretation for the approaching motion of M31 is that it is due to pure initial conditions, and is unrelated to the mass of the Local Group.

Van der Bergh suggested that our Galaxy and M31 might not form any coherent system, and that both galaxies "were passing each other as ships pass in the night" (Lynden-Bell, 1983).

b) The "timing" argument of Kahn and Woltjer.

The most widely accepted interpretation of the negative velocity of M31 was first given by Kahn and Woltjer (1959). They assumed that this double system has negative energy, i.e. it is held together by gravitational forces. However, considering visible matter only, they estimated the kinetic energy of the system to be about 1.25×10^{58} erg, and the gravitational energy -6×10^{57} erg. Even with an apparent positive energy (unbounded system) they considered the possibility of large quantities of intergalactic material in the form of gas, which would render the total energy negative. This gaseous intergalactic mass was not confirmed by later observations. Instead, today, the argument of Kahn and Woltjer is considered as a proof for either the existence of large dark matter halos surrounding M31 and the Milky Way or (at least) a large common DM super halo pervading the Local Group. They deduced, with a simple order of magnitude argument, that the effective mass was larger than $1.8 \times 10^{12} M_{\odot}$, about six times larger than the reduced mass of M31 and the Milky Way. Lynden-Bell (1983) has presented a more precise description.

It is interesting to note, also in this historic paper, that Kahn and Woltjer (1959) considered that the ram pressure produced by this hypothetical intergalactic gas, due to the motion of both galaxies with respect to it, was responsible for warps of both galaxies. This hypothesis for the origin of warps has today been largely forgotten, but it could explain the coherence in the orientation of the warps of M31, M33 and the Milky Way shown by Zurita and Battaner (1997). This coherence can only be explained by the hypothesis of Kahn and Woltjer and by the magnetic hypothesis (Battaner, Florido and Sanchez-Saavedra, 1990, 1991; Battaner, 1995; Battaner, Florido, 1997; Battaner and Jimenez-Vicente, 1998; Battaner et al. 1991; see also Binney, 1991, and Kuijken, 1997).

Coming back to the "timing" argument, let us obtain a similar order of magnitude, by an argument closer to that presented by Lynden-Bell (1983). Suppose that the pregalaxies later to become M31 and the Milky Way were formed at Recombination. Inhomogeneity seeds were previously developed, but at Recombination, photon decoupling allowed matter to freely collapse. Identifying Recombination as the epoch of the Local Group birth, at about 10^6 years after the Big Bang, is equivalent to this birth being produced at the very beginning of the Universe, as 10^6 years is negligible when compared with 14 Gyr, at present.

Then the Universe was much smoother, so we can assume a vanishing initial transverse velocity. The Local Group, i.e. the two galaxies, were born so close to each other that gravitation was stronger than the expansion effect, so that we assume that during the period of the birth of both galaxies, there was a negligible relative velocity between them, in the line connecting them. Therefore, we assume that 14 Gyr ago, both galaxies were at rest with respect to each other, and since then their mutual gravitational attraction has reduced their separation and is responsible for the 125 kms^{-1} approaching velocity observed today.

The general equations for the orbit in the framework of Newtonian Mechanics adopt the following parametric form

$$r = a(1 - \epsilon \cos \eta) \tag{31}$$

$$\Omega t = \eta - \epsilon \sin \eta \tag{32}$$

where r is the mutual distance, t the time and ϵ the eccentricity, while Ω and a are constants. The parameter η is called the eccentric anomaly. The sum of the masses of both galaxies, M, is related to these constants, through

$$GM = \Omega^2 a^3 \tag{33}$$

If ϵ were zero, we would have $r = a$ (constant) and $\eta = \Omega t$, e.g. a circular orbit with a constant velocity. But given that our initial transverse velocity was assumed to be null, our orbit cannot be circular, but rather, it will become approximately a straight line. We thus consider $\epsilon = 1$.

Figure 13 presents various possibilities:

the first possibility provides the lowest mass and we will concentrate on this one. We have

$$r = a(1 - \cos\eta) \tag{34}$$

$$\Omega t = \eta - \sin\eta \tag{35}$$

therefore

$$\dot{r} = a \sin\eta\dot{\eta} \tag{36}$$

$$\Omega = (1 - \cos\eta)\dot{\eta} \tag{37}$$

At the birth (approximately, at the Big Bang) we set $t = t_1$. Then, $\dot{r}_1 = 0$, as we have assumed. $\dot{\eta} \neq 0$, always, as otherwise (37) would imply $\Omega = 0$. Therefore, $\sin\eta_1 = 0$, which gives either $\eta_1 = 0$ or $\eta_1 = \pi$. But $\eta_1 = 0$ would imply $r_1 = 0$, while we have started with the distance of the galaxies being a maximum ($2a$). Therefore, $\eta_1 = \pi$. Hence, $r_1 = 2a$ (as expected), $\Omega t_1 = \pi$, $\dot{r}_1 = 0$, $\Omega = 2\dot{\eta}_1$.

At the present time, we set $t = t_2$. Then

$$r_2 = a(1 - \cos\eta_2) \tag{38}$$

$$\Omega t_2 = \eta_2 - \sin\eta_2 \tag{39}$$

Then

$$\Omega t_2 - \Omega t_1 = \Omega(t_2 - t_1) = T\Omega = \eta_2 - \sin\eta_2 - \pi \tag{40}$$

because $t_2 - t_1 = 14$, if we adopt 1 Gyr as time unity. We know $r_2 = 650$ (taking 1 kpc as distance unity). We also know $\dot{r_2} = -125$ (if we adopt 1 kpc/Gyr as unity for the velocity; 1 km/s $\approx$ 1 kpc/Gyr !)

$$\dot{r}_2 = a \sin\eta_2\dot{\eta}_2 \tag{41}$$

$$\Omega = (1 - \cos\eta_2)\dot{\eta}_2 \tag{42}$$

Eliminating $\dot{\eta}_2$

$$\dot{r}_2 = \frac{a \sin\eta_2 \Omega}{1 - \cos\eta_2} \tag{43}$$

With (16) and (43)

$$\frac{\dot{r}_2}{r_2} = \frac{\sin\eta_2 \Omega}{(1 - \cos\eta_2)^2} \tag{44}$$

and taking the value of Ω given by (40)

$$\frac{\dot{r}_2}{r_2} = \frac{\sin\eta_2}{(1 - \cos\eta_2)^2}\frac{\eta_2 - \sin\eta_2 - \pi}{T} \tag{45}$$

Defining, $\varphi_2 = \eta_2 - \pi$

$$\frac{\dot{r}_2}{r_2} T = \frac{\varphi_2 \sin\varphi_2 + \sin^2\varphi_2}{(1+\cos\varphi_2)^2} \tag{46}$$

Taking the numerical values for r_2, $\dot{r}_2$ and T, the solution of this equation, approximately, gives $\varphi_2 = 1.59$, $\eta_2 = 4.73$. Hence (with (40)), $\Omega = 0.18 Gyr^{-1}$. Therefore

$$\Omega t_2 = \Omega t_1 + 14\Omega = \pi + 14 \times 0.18 = 5.66 \tag{47}$$

$$t_2 = 31 Gyr \tag{48}$$

(Note that the time of the Big Bang is $t_1 = 17 Gyr$. We are not taking the Big Bang as the origin of time!). Then, with (38), we have:

$$a = 662 kpc \tag{49}$$

In our modest calculation, at the beginning both galaxies were $2a = 1324$ kpc apart and they were at rest. Now they are 650 kpc apart (about half the initial distance) and they are approaching at 125 km/s.

With all these values, we deduce for the mass of the pair of galaxies

$$M \sim 2 \times 10^{12} M_\odot \tag{50}$$

which is clearly much more than the visible mass of the pair of about $5 \times 10^{11} M_\odot$. Despite the long calculation, the order of magnitude is just given by $M = V^2 r/G$, where r and V are the distance and velocity of M31.

c) In the above argument we considered two mass points with mutual attraction, but the dark matter apparently encountered may be distributed in a single extended halo. If the force of gravity acting on the Galaxy were due to this Local Group super-halo, the equation to be integrated would be

$$\ddot{r} + \frac{4}{3}\pi G \rho r = 0 \tag{51}$$

where ρ is the density of the intergalactic medium, which, for simplicity we assume to be constant. In this case the angular velocity of the periodic motion would be

$$\Omega = \left(\frac{4}{3}\pi G \rho\right)^{1/2} \tag{52}$$

We can, as before, obtain detailed values of Ω and the initial distance between the new born Milky Way and the centre of the Local Group, identified with the position of M31. In this case (r=a, $\dot{r} = 0$ at $t = 0$; the origin of time is now the Big Bang, approximately. Now, a is the maximum separation of the Milky Way, instead of $2a$, as in the previous case). We adopt $r = 650 kpc$, $\dot{r} = -125 kpc/Mpc$, $T = 14 Gyr$ as before,

$$r = a \cos \Omega t \tag{53}$$

$$\dot{r} = -a\Omega \sin \Omega t \tag{54}$$

Dividing the formulae

$$\frac{\dot{r}}{r} = -\Omega T \tag{55}$$

hence

$$\Omega = 0.083 Gyr^{-1} \tag{56}$$

and

$$a = 1633 kpc \tag{57}$$

For the density of dark matter in the Local Group, we obtain

$$\rho = 2.7 \times 10^{-29} gcm^{-3} \tag{58}$$

This value is much lower than the minimum value estimated by Kahn and Woltjer (about $1.6 \times 10^{-28} gcm^{-3}$) and slightly higher than the critical density to close the Universe ($\sim 10^{-29} gcm^{-3}$). The common halo hypothesis is not easy to reject.

d) The Local Group, rather than two main galaxies and several satellites together with some minor members, should be considered as a primordial inhomogeneity which has only recently collapsed to form its present galactic members. Like any other inhomogeneity it has evolved through the radiation dominated epoch with $\delta = \Delta\rho/\rho \propto R$, decaying transverse velocities and increasing radial velocities in a moderate collapse. Then inhomogeneities reached an acoustic epoch, which for masses typical of the Local Group began at $z = 10^5$ approximately (see later, Fig. 22). After the Recombination epoch the Local Group pursued its process of collapse with the relative density contrast increasing as R, where R is the cosmological scale factor, the transverse velocities decreasing as R^{-1} and - what is most important for our purposes- the radial velocities increasing as $R^{1/2}$. After that, the collapse became non-linear and these variations with the cosmic scale factor became complicated and faster. As $\delta \gg 1$ we find ourselves in the non-linear regimen, but we will consider a linear evolution to find typical orders of magnitude. In this picture a naive formula relating the present velocity V_0 of an inhomogeneity with present size λ_0 and actual relative density contrast is (Battaner, 1996)

$$V_0 = H_0 \lambda_0 \delta_0 \tag{59}$$

If the Milky Way and M31 were condensations within the Local Group, V_0 would be identified with the relative velocity between these two galaxies, with λ_0 and δ_0 being typical parameters characterizing the size and the density contrast of the Local Group.

This interpretation of the negative recession velocity of M31 is fully compatible with the scenario of an approach between the two galaxies within an expanding universe but somewhat in contrast with present hierarchical models, in which small structures form first, which will be accounted for later. As the velocities, before Recombination, do not reach high values (Florido and Battaner, 1997) we can start our calculations at Recombination. From the above formula, taking $V_0 \sim 125km/s$, $H_0 = 60km/(sMpc)$ and $\lambda_0 \sim 0.65Mpc$, we obtain $\delta_0 \approx 5.5$. Then

$$\rho_0 = 5.5 = \frac{\rho_{LocalGroup} - < \rho >}{< \rho >} \tag{60}$$

where $< \rho >$ is the average density in the Universe. Hence, for the Local Group

$$\rho_{LG} = (\delta + 1) < \rho > \tag{61}$$

Let us adopt for $< \rho >= 0.3 \times 10^{-29} gcm^{-3}$, thus obtaining

$$\rho_{LG} \cong 2 \times 10^{-29} gcm^{-3} \tag{62}$$

Let us compare the different results. Methods c) and d) give a similar order of magnitude, about $2.7 \times 10^{-29} gcm^{-3}$. The mass corresponding to this density depends on the volume. The density surely decreases outwards. Suppose a moderate equivalent radius of 650 kpc; then the mass of the Local Group would be $4 \times 10^{11} M_\odot$, which is approximately the visible mass. Or suppose an equivalent radius of 1 Mpc. In this case, we obtain $1.5 \times 10^{12} M_\odot$, in reasonable agreement with method a).

Not only should the results be compared, but also the basic formulae when the numerical coefficients close to unity are ignored. Essentially, methods b) and c) use $M \approx V^2 r/G$, where V is the approaching velocity of M31 and r its distance. Of course, the more detailed arguments presented provide a more precise result, but which cannot greatly differ from this value ($2.3 \times 10^{12} M_\odot$). However, method d) is quite different. The order-of-magnitude lying behind the calculation is of the type $M \approx \frac{Vr^2}{H_0} < \rho >$. In a critical Universe $< \rho >= 3H_0^2/8\pi G$. This method is not intrinsically related to the other two. The orders of magnitude coincide because, curiously, V/r is of the order of H_0. In most pairs the orbital period is of the order of H_0^{-1}.

Summarizing, unless M31 and the Milky Way are like "ships passing in the night" (a possibility that cannot be totally disregarded), the Local Group seems to have 4 times more mass than we see as stellar light. But we don't know where this mass lies, whether in galactic dark matter halos or in a large common super halo. The difficulties encountered in the interpretation of the closest binary system are translated to the interpretation of other binary systems.

3.5.2 Statistics of binary galaxies

It is tempting to observe binary galaxies to obtain galactic masses. Typical periods are, however, so large that orbits cannot be observed. On the other hand, typical distances between the two galaxies are much larger than visible galactic sizes, and therefore we could, in principle, obtain total masses.

The observations permit the obtention of "projected distances", Δr, and of "differences in the velocity component along the line-of-sight", Δv. From both series of data we must infer a mean M/L ratio. The analysis must be statistical as no parameter of the individual orbits is known.

The first problem to resolve, and a very serious one, is the selection of a suitable sample. Chance superpositions must be avoided: including in the sample two unbounded galaxies, for which the velocity difference is due to Hubble's flow, could give a very high Δv and hence an overestimation of the mean mass. If the pair is not isolated, the influence of a third galaxy could produce a misinterpretation of the results; galaxies are often in small or large clusters and are rarely found in truly isolated pairs.

Usually, only pairs with a projected separation of less than a given value, R, are accepted in the sample. Binney and Tremaine (1987) have warned of this danger. Suppose that we take R as smaller than the mean true pair distance. Then, the velocity should be mainly perpendicular to the line-of-sight, and therefore much greater than the projected velocity along the line of sight; then Δv would be underestimated, as would the galactic masses.

Limit values of Δr and of Δv become necessary, but then, we find the results that we expect. Indeed, Sharp (1990) after comparing the large discrepancies between different workers, even with similar samples, was very pessimistic about the ability of these statistical approaches to derive galactic masses.

To interpret statistical distributions of Δr and Δv in order to obtain M, the mass of the two galaxies, it is necessary to adopt a law for the distribution of the true separation r; for example, $\varphi(r) \propto r^{-\gamma}$, deduced with the two-point correlation function of galaxies (Peebles, 1974). Observations indicate that $\gamma \sim 1.8$. This distribution might not be valid for close binary systems. It is also necessary to adopt a hypothesis about the orbits, and more precisely about the value of the parameter β, defined as

$$\beta = 1 - \frac{< v_\theta^2 >}{< v_r^2 >} \tag{63}$$

where v_θ and v_r are the azimuthal and radial components of the velocity. If the orbits are radial, $< v_\theta^2 >= 0$ and we should take $\beta = 1$. If the orbits are perfectly circular, then $< v_r^2 >= 0$ and $\beta = -\infty$. An interesting intermediate assumption is the condition of isotropy $< v_r^2 >=< v_\theta^2 >$, hence $\beta = 0$.

A natural way to study binary samples is the adoption of galaxies as mass points. In a classical analysis by White et al. (1983), however, it was demonstrated that the mass point model does not fit the data. This model predicts a correlation between Δv and Δr (clearly $(\Delta v)^2$ should correlate with $(\Delta r)^{-1}$), which is not found. This negative result is highly interesting, as it can be interpreted as being due to the existence of greatly extended halos. If the force, instead of $-GM/r^2$, were of the type $\propto r^{-1}$, the correlation $[(\Delta v)^2 \leftrightarrow (\Delta r)^{-1}]$ would not exist.

This classical paper also claimed other evidence favouring the existence of massive extended halos. They found dark-to-luminous mass ratios higher than those found with rotation curves. This is really to be expected, because mass determinations from rotation curves are made at a maximum radius lower than the rim of the halo, while a companion could be far away, at a distance greater than the sum of both halos. Indeed their results are compatible with extrapolations of observed rotation curves. These authors found a relation of the type $M/L \propto L^{-3/4}$. Therefore, low luminosity galaxies should contain large amounts of dark matter. Lake and Schommer (1984) confirmed very high M/L values in a sample of dwarf irregular pairs.

However, Karachentsev (1983, 1985) found no evidence of dark matter in binary systems, using very large samples, even containing some galaxies with well observed flat rotation curves.

Honma (1999) found M/L for spiral pairs in the range 12-16, lower than M/L for ellipticals, confirming previous results by Schweizer (1987). These values are clearly lower than those previously reported.

Among the large list of workers who have attempted to obtain proof of dark matter with this method, noticeable are the studies by van Moorsel (1987), Charlton and Salpeter (1991) and others, favouring the scenario of a large common dark matter envelope, as we have seen in the pair formed by M31 and the Milky Way. From the cosmological point of view, whether dark matter lies in individual or in common halos is unimportant, but from the point of view of galactic structure and evolution, the two models are completely different.

As stated by Binney and Tremaine (1987) "the mass-to-light ratio of binary galaxies is probably large, but not so large as the ratio of the mass of papers on this subject to the light they have shed on it". Even the concept of binary systems is controversial: with typical velocities of 100 kms^{-1} and separations of 100 kpc, a typical value for the orbital period is of the order of Hubble's time. In most cases, such as in the M31-Milky Way pair, a simple orbit has not been completed. The Universe is expanding with a typical time of the orbital period. Nevertheless, despite the large variety of results, it should be emphasized that the most widely accepted point of view is that binary galaxies possess large amounts of dark matter, either in individual halos or in common super halos.

3.6 Globular clusters and satellites

Globular clusters and satellite galaxies are stellar components at large distances from the centre and are therefore ideal probes of the gravitational potential of a galaxy. The Milky Way may constitute, once more, the best example. Actually, globular clusters are typically at distances of less than 10 kpc, much lower than typical halo sizes, and are therefore not so suitable. Satellite galaxies are further away but they are statistically scarce, probably only nine satellites belonging to our Galaxy. However, satellites are also observable in other galaxies.

Suppose a Keplerian potential $-GM/r$ and that, therefore, for satellites with elliptical orbits

$$r = a(1 - e\cos\eta) \tag{64}$$

$$\omega t = \eta - e\sin\eta \tag{65}$$

as in 3.5.1. Time derivatives are

$$\dot{r} = -ae\sin\eta\dot{\eta} \tag{66}$$

$$\omega = (1 - e\cos\eta)\dot{\eta} = \frac{r}{a}\dot{\eta} \tag{67}$$

Therefore

$$\dot{r}^2 r = \omega a^3 e^2 \sin\eta^2\dot{\eta} \tag{68}$$

Let us calculate the mean time value of this quantity during a complete orbit

$$< \dot{r}^2 r >= \frac{\omega}{2\pi}\int_0^{2\pi} \omega a^3 e^2 \sin\eta^2 \frac{d\eta}{dt} dt = \frac{\omega^2}{2\pi} a^3 e^2 \int_0^{2\pi} \sin^2\eta d\eta = \frac{1}{2}\omega^2 a^3 e^2 = \frac{1}{2}GMe^2 \tag{69}$$

Therefore, the galactic mass M could be obtained from the quantity $< \dot{r}^2 r >$. If we now consider not a single cluster or satellite but an assemblage of them in a given time, the mean time value would not differ from the mean value of $\dot{r}_i^2 r_i$ of the different clusters, at present. We need to know the eccentricity, which would be different for the different clusters. The mean-square value for isotropic orbits is 1/2. Binney and Tremaine (1987), who have presented this argument, propose

$$M = \frac{4}{G} < \dot{r}^2 r > \tag{70}$$

This method was applied by Lynden-Bell et al. (1983), who obtained $M = 3.8 \times 10^{11} M_\odot$ with 9 satellites of the Milky Way (both Magellanic Clouds, Leo I and II, Fornax, Sculptor, Ursa Minor, Draco and Carina).

The adoption of a Keplerian potential was not fully justified as 6 out of the 9 selected satellites are at distances of less than 100 kpc, lower than a reasonable size of the dark matter halo. The unknown value of the eccentricities is also a major source of errors. If the orbits were radially elongated, with $e = 1$, the calculated mass would be closer to the galactic mass deduced from the visible matter alone. That orbits could be elongated rather than isotropic is somewhat suggested by the fact that (with the exceptions of Leo I and Leo II, which are too far away) all the satellites of the Milky Way are roughly aligned in a line connecting ($b = -45^o$, $l = 270^o$) $\rightarrow$ ($b = 45^o$, $l = 90^o$).

Ashman (1992) and Trimble (1987) have summarized previous work carried out by Little and Tremaine (1991), Zaritsky et al. (1989), Salucci and Frenk

(1989), Peterson and Latham (1989), Kulessa and Lynden-Bell (1992) and others. The results obtained by the different authors are very different, depending on the inclusion or exclusion of some distant satellites, in particular on the inclusion or exclusion of Leo I. However, a mass of $10^{12} M_\odot$ and a halo radius of 100 kpc are typical values.

This type of analysis will be more promising in the future, when proper motions of the satellites of the Milky Way become available. Wilkinson and Evans (1999) have incorporated into the computation the known proper motions of 6 satellites. They obtain a value of about $2 \times 10^{12} M_\odot$ for the mass of the Milky Way, with the inclusion or exclusion of Leo I not being so important as when only radial motions are considered. The results are model dependent and these authors have chosen a peculiar one, called the "truncated flat" rotation curve, in which the density decreases as r^{-2} in the inner parts and decreases as r^{-5} in the outer ones.

Tidal radii (see Section 3.3) of globular clusters and satellite galaxies have also been considered (e.g. Innanen et al. 1983) but these radii are obtained by an extrapolation of the photometric data, which introduces a lot of uncertainty. A mass of about $9 \times 10^{11} M_\odot$ and a halo size of at least 44 kpc were obtained by these authors.

Satellites of other galaxies have been studied by Zaritsky et al. (1993), Zaritsky and White (1994), Zaritsky (1997) and others. The problem has much in common with that of binary galaxies but presents particular interest because, as satellites are supposed to be low mass systems they can be considered as test particles orbiting in the total mass of the primary. The above authors have observed 115 satellites around 69 isolated primary galaxies. They conclude that DM halos do exist and that they extend to distances of over 400 kpc, actually a very large figure, but many characteristic facts are difficult to explain: a) There is a complete lack of correlation between Δv and Δr, which impedes the obtention of the mean value of galaxies. b) There is a complete lack of correlation between Δv and the HI widths. It has been mentioned that Salucci and Persic (1997) and, as will be mentioned later, theoretical models do not expect a large correlation between these two quantities, but not a vanishing one, either. c) Satellites seem to be preferentially distributed near the plane perpendicular to the rotation axis of the primary. d) The assemblage of all the satellites seems to present a "rotation curve" around a typical primary without signs of becoming flat. Note that the possibility of a common DM halo without an internal DM structure is a picture compatible with these observations.

4 Theory

The theoretical interpretation of rotation curves is one of the goals of the so called CDM (Cold Dark Matter) hierarchical models of formation and evolution of galaxies, which are at present the most widely accepted models. Other explanations, such as the MOND (Modified Newtonian Dynamics) and the magnetic models, are tentative and will also be mentioned.

4.1 The nature of galactic dark matter

Galaxies are born out of primordial fluctuations with an evolution probably driven by gravitation as the dominant effect. Gravitation, as a geometric concept, has the same effect on the different types of particles. Some forces other than gravitation, such as the interaction with photons, dissipative effects, magnetic fields, etc., could also have an influence and act on the involved particles differentially, but an overall trend for galaxies and clusters to have a similar composition to the general composition of the Universe is to be expected.

Our knowledge about the composition of the Universe has changed in recent times with respect to the classical view, summarized, for instance, by Schramm (1992). This new conception has been reviewed, for instance, by Turner (1999a,b). The dominant matter is considered to be cold dark matter (CDM), consisting of particles moving slowly, so that the CDM energy density is mainly due to the particle's rest mass, there being a large series of candidates for CDM particles, but axions and neutralinos being the most attractive possibilities.

Big Bang nucleosynthesis studies have been able to accurately determine the baryon density as $(0.019 \pm 0.0012)h^{-2}$. The cluster baryon density has also been accurately determined by X-ray and the Sunyaev-Zeldovich effect to be $f_B = (0.07 \pm 0.007)h^{-3/2}$ and, assuming that rich clusters provide a fair sample of matter in the Universe, also $\Omega_B/\Omega_M = f_B$, from which, it follows $\Omega_M = (0.27 \pm 0.05)h^{-1/2}$. The Universe is however flat, $\Omega = 1$, with the CMB spectrum being a sensitive indicator. Therefore $\Omega = 1 = \Omega_M + \Omega_\Lambda$, where $\Omega_\Lambda \sim 0.7$ represents the contribution of the vacuum energy, or rather, the contribution of the cosmological term Λ. With this high value of Ω_Λ the Universe should be in accelerating expansion, which has been confirmed by the study of high-redshift supernovae, which also suggest $\Omega_\Lambda \sim 0.7$ (Perlmutter, Turner and White, 1999; Perlmutter et al. 1999). The stellar or visible matter is estimated to be $\Omega_V = 0.003 - 0.006$. All these values can be written in a list easier to remember, with values compatible with the above figures, adopting the values of $H_0 = 65 km s^{-1} Mpc^{-1}$; h=0.65:

$$\Omega_V = 0.003$$

$$\Omega_B = 0.03$$

$$\Omega_M = 0.3$$

$$\Omega = 1$$

less precise but useful for exploratory fast calculations.

A large cluster should have more or less this composition, including the halo of course, even if a halo could contain several baryonic concentrations or simply none. Therefore, a first direct approach to the problem suggests that halos are non baryonic, with baryonic matter being a minor constituent.

This is also the point of view assumed by most current theoretical models (this will be considered later, in Section 4.2.2), which follow the seminal papers by Press and Schechter (1974) and White and Rees (1978). We advance the comment that, in these models, a dominant collisionless non dissipative cold dark matter is the main ingredient of halos while baryons, probably simply gas, constitute the dissipative component, able to cool, concentrate, fragment and star-producing. Some gas can be retained mixed in the halo, and therefore halos would be constituted of non-baryonic matter plus small quantities of gas, its fraction decreasing with time, while mergers and accretion would provide increasing quantities to the visible disks and bulges. Therefore, a first approach suggests that galactic dark matter is mainly non-baryonic, which would be considered as the standard description. Baryons, and therefore visible matter, may not have condensed completely within a large DM halo, and therefore the baryon/DM ratio should be similar in the largest halos and in the whole Universe, although this ratio could be different in normal galaxies.

However, other interesting possibilities have also been proposed. The galactic visible/dark matter fraction depends very much on the type of galaxy, but a typical value could be 0.1. This is also approximately the visible/baryon matter fraction in the Universe, which has led some authors to think that the galactic dark matter is baryonic (e.g. Freeman, 1997) in which case the best candidates would be gas clouds, stellar remnants or substellar objects. The stellar remnants present some problems: white dwarfs require unjustified initial mass functions; neutron stars and black holes would have produced much more metal enrichment. We cannot account for the many different possibilities explored. Substellar objects, like brown dwarfs, are an interesting identification of MACHOs, the compact objects producing microlensing of foreground stars. Alcock et al. (1993), Aubourg et al. (1993) and others have suggested that MACHOSs could provide a substantial amount of the halo dark matter, as much as 50-60% for masses of about 0.25 $M_\odot$, but the results very much depend on the model assumed for the visible and dark matter components, and are still uncertain. Honma and Kan-ya (1998) argued that if the Milky Way does not have a flat rotation curve out to 50 kpc, brown dwarfs could account for the whole halo, and in this case the Milky Way mass is only $1.1 \times 10^{11} M_\odot$.

Let us then briefly comment on the possibility of dark gas clouds, as defended by Pfenniger and Combes (1994), Pfenniger, Combes and Martinet (1994) and Pfenniger (1997). They have proposed that spiral galaxies evolve from Sd to Sa, i.e. the bulge and the disk both increase and at the same time the M/L ratio decreases. Sd are gas-richer than Sa. It is then tempting to conclude that dark matter gradually transforms into visible matter, i.e. into stars. Then, the dark matter should be identified with gas. Why, then, cannot we see that gas? Such a scenario could be the case if molecular clouds possessed a fractal structure from 0.01 to 100 pc. Clouds would be fragmented into smaller, denser and colder sub-clumps, with the fractal dimension being 1.6-2. Available millimeter radiotelescopes are unable to detect such very small clouds. This hypothesis would also explain Bosma's relation between dark matter and gas (Section 2.3), because dark matter would, in fact, be gas (the observable HI disk could be the observable atmosphere of the dense molecular clouds). In this case, the dark matter should have a disk distribution.

The identification of disk gas as galactic dark matter was first proposed by Valentijn (1991) and was later analyzed by González-Serrano and Valentijn (1991), Lequeux, Allen and Guilloteau (1993), Pfenniger, Combes and Martinet (1994), Gerhard and Silk (1996) and others. H_2 could be associated to dust, producing a colour dependence of the radial scale length compatible with large amounts of H_2. Recently, Valentijn and van der Werf (1999) detected rotational lines of H_2 at 28.2 and 17.0 μm in NGC 891 on board ISO, which are compatible with the required dark matter. If confirmed, this experiment would be crucial, demonstrating that a disk baryonic visible component is responsible for the anomalous rotation curve and the fragility of apparently solid theories. Confirmation in other galaxies could be difficult as H_2 in NGC 891 seems to be exceptionally warm (80-90 K).

A disk distribution is, indeed, the most audacious statement of this scenario. Olling (1996) has deduced that the galaxy NGC 4244 has a flaring that requires a flattened halo. However, this analysis needs many theoretical assumptions; for example, the condition of vertical hydrostatic equilibrium requires further justification, particularly considering that NGC 4244 is a Scd galaxy, with vertical outflows being more important in late type galaxies. Warps have also been used to deduce the shape of the halo. Again, Hofner and Sparke (1994) found that only one galaxy NGC 2903, out of the five studied, had a flattened halo. In this paper, a particular model of warps is assumed (Sparke and Casertano, 1988), but there are other alternatives (Binney 1991, 1992). The Sparke and Casertano model seems to fail once the response of the halo to the precession of the disk is taken into account (Nelson and Tremaine, 1995; Dubinski and Kuijken, 1995). Kuijken (1997) concludes that "perhaps the answer lies in the magnetic generation of warps" (Battaner, Florido and Sanchez-Saavedra 1990). On the other hand, if warps are a deformation of that part of the disk that is already gravitationally dominated by the halo, the deformation of the disk would be a consequence of departures from symmetry in the halo. To isolate disk perturbations embedded in a perfect unperturbed halo is unrealistic. Many other proposals have been made to study the shape of the halo, most of which are reviewed in the cited papers by Olling, and in Ashman (1982), but very different shapes have been reported (see section 3.4).

There is also the possibility that a visible halo component could have been observed (Sackett et al. 1994; Rausher et al. 1997) but due to the difficulty of working at these faint levels, this finding has yet to be confirmed.

Many other authors propose that the halo is baryonic, even if new models of galactic formation and evolution should be developed (de Paolis et al. 1997). This is in part based on the fact that all dark matter "observed" in galaxies and clusters could be accounted for by baryonic matter alone. Under the interpretation of de Paolis et al. (1995) small dense clouds of H_2 could also be identified with dark matter, and even be responsible for microlensing, but instead of being distributed in the disk, they would lie in a spherical halo.

4.2 CDM theoretical models

Theoretical models of galaxies consider their origin and evolution. It is difficult to review the early history of these theories and identify which of them have had a decisive influence on our present ideas. Most contemporary theories about the origin of galaxies are based on three decisive papers by Press and Schechter (1974), White and Rees (1978) and Peebles (1982), which will be commented on later, and have in common the hypothesis that the dark matter is cold (CDM) and that, at a given time, CDM halos arose through a hierarchy of different sized halos formed from mergers of smaller halos. At least four steps characterize the evolution of a galaxy:

a) Small density fluctuations, probably originated by quantum fluctuations before the epoch of Inflation or at cosmological phase transitions, grow during the radiation dominated universe and provide a fluctuation spectrum after the epoch of Recombination.

b) CDM overdensities accrete matter and merge. The hierarchical formation of greater and greater halos produces the present galactic and cluster structures.

c) Baryons cool and concentrate at the centre of halos and constitute the visible component of galaxies. The explanation of the Hubble sequence and the origin of rotation of galaxies would be goals of the study of this phase.

d) Once the basic structure of a galaxy with its different components has been established, it is necessary to follow its evolution due to star formation, gas ejected from stars, progressive metal enrichment, matter flows connecting the intra and extra media, small internal motions, etc.

4.2.1 Growth of primordial fluctuations

Suppose we start our analysis shortly after Annihilation. Then, a primordial energy density fluctuation spectrum must be assumed. One of the most simple hypotheses is the spectrum of Harrison and Zeldovich which is rest mass independent and which arises naturally from the quantum fluctuation at Inflation, but there are other more exotic possibilities; indeed, the spectrum has been characterized by some parameters which are considered free in some numerical calculations. The subsequent evolution is a consequence of this initial spectrum and of the nature of the matter, mainly through the equation of state.

Most models do not explicitly consider this first phase. It is considered that an unknown primordial density fluctuation spectrum is responsible for an unknown post-Recombination spectrum and this, therefore, is equivalent to assuming the initial spectrum after Recombination and this complicated phase is thus avoided. We consider this procedure somewhat dangerous because even if the initial spectrum is random some regular structure may be inherited after Recombination. For example, primordial magnetic fields may be responsible for very large scale filaments ($\sim$ 100 Mpc) as discussed later. Moreover, the existence of periodic structures forming a lattice, actually observed whatever the cause may be, must be understood to assess how CDM halos merge at later epochs. These points will be addressed later.

As in the case of stellar collapses, the basic concept is Jeans' Mass. We must know which masses are able to collapse and how the collapse grows as a function of time. Both phenomena depend on the epoch during the thermal history of the Universe. The basic treatment was developed by Lifshitz (1946), Zeldovich (1967) and Field (1974) and has been clearly incorporated in the well-known book by Weinberg (1972). Some more recent books also address this analysis (Kolb and Turner, 1990; Battaner, 1996).

The protogalactic collapse has some differences with respect to the protostellar collapse, mainly:

a) Protostellar collapses are considered to be isothermal, because photons are able to quit the protostellar cloud freely and the temperature remains constant. It is then obtained for Jeans' Mass, $M_J \propto \rho^{1/2}$. The fact that M_J, the minimum mass able to collapse, increases when the collapse proceeds produces the fragmentation of the cloud until the smaller fragments are so dense that the isothermal regime breaks down. The pre-Recombination collapse involves clouds made up of CDM particles, baryons and, mainly, photons. Photon clouds have no way to remain isothermal when they contract. Adiabatic collapses are to be assumed, which does not lead to any fragmentation.

b) Contraction within the expansion. During the collapse, the dimensionless quantity δ, defined as $\delta = (\rho - <\rho>)/<\rho>$ (where ρ is the inhomogeneity density and $<\rho>$ its mean value in the Universe), increases, but as $<\rho>$ decreases because of the general expansion, ρ need not necessarily increase. The collapse is relative. Indeed, present densities in a galaxy are greater than, but comparable to, densities before the collapse. As a zeroth-order language, isolation rather than absolute contraction gives rise to galaxies. The time evolution of δ, i.e. of the relative overdensity, provides a simpler description. The effect of expansion is not at all negligible, because the characteristic time of expansion, 1/H, is of the order of the period of Jeans' wave, λ_J/V_s, where λ_J is Jeans' wavelength and V_s the speed of sound, with both being variable during history of the Universe.

From the point of view of the physics involved, pre-Recombination collapses require a general-relativistic treatment as they are fluctuations in a very hot medium (photons) and the curvature they produce is not only non-ignorable but a dominant effect.

Jeans' Mass is calculated to be $M_J \propto R^3$ during the era between Annihilation of electrons and positrons and the transition epoch dividing the radiation and matter dominations; R is the cosmological scale factor. Between this last epoch (i.e. Equality) and Recombination, Jeans' Mass increases to a constant asymptotic value, $M_J \approx 4 \times 10^{19} M_\odot$, which is never reached because, at Recombination, the scenario abruptly changes, with a sudden fall from about $10^{17} M_\odot$ to about $10^5 M_\odot$. In the post-Recombination era $M_J \propto R^{-3/2}$. The complete function $M_J(R)$ is depicted in Fig. 14.

In this picture, we may follow the stability of an inhomogeneity with a rest mass of $10^{12} M_\odot$, a typical value of the galactic mass, dark matter included. Its mass is in principle higher than Jeans' Mass, and therefore we initially find this protogalaxy in a collapsing phase. The collapse is not so fast, as we will see later, and is truncated when $R/R_0 \sim 10^{-5}$ approximately. The proto-galaxy then enters a stable state and Jeans' wave just produces acoustic oscillations. There is not much time to oscillate in this Acoustic era, less than one complete period, because the Recombination sudden falls, leading our homogeneity to unstable conditions again. In other words, once baryons are no longer coupled to photons they are free to collapse.

CDM particles may alter this picture if they have no interaction with photons, as they are free to collapse when they become dominant. They then create potential wells where, after Recombination, the baryons fall. In this case the Acoustic era would be absent.

In the same way that the study of Jeans' waves provides the value of typical stellar masses, it would be desirable to obtain typical values of masses of galaxies and also of clusters and superclusters, because the analysis mentioned considers any inhomogeneity. A large enough mass would always collapse, but we could expect at least a minimum value of collapsed systems.

If the dominant matter particles were baryons, or any other type of particles interacting with photons, then damping by non-perfect fluid effects would affect the oscillations in the Acoustic Era, therefore preventing small mass inhomogeneities from reaching Recombination. The mechanism of *photon diffusion* is of this type. The fast photons would tend to escape from the overdensity cloud and then push baryons outwards, via the interaction due to Thomson scattering. This is equivalent to a viscosity and a heat conduction, which are expected to be important when the photon mass free time is of the order of the cloud size. The so called Silk Mass is calculated is such a way. Numerical estimations provide values of the Silk Mass of the order of $10^{12} M_\odot$, a very significant value. However, we will see that the model of CDM hierarchical structure formation considers a different scenario, with high masses only being limited by the finite time of the Universe. Even if a mechanism similar to photon diffusion had been at work before Recombination, much smaller masses, much lower than $10^{12} M_\odot$, would be the components of the initial merging CDM blocks. The smaller galaxies are of the order of $10^7 - 10^8 M_\odot$.

Another mechanism, called *Free Streaming*, would give a lower limit to collapsing clouds, if the DM particles were hot. Suppose they were neutrinos, for example; in this case, they would escape from the initial inhomogeneity if this homogeneity were small. When the expansion proceeds and the temperature of the Universe is low enough, the neutrino speed becomes small, which limits the distance a neutrino is able to run. When $kT \sim m_\nu c^2$, where m_ν is the neutrino's mass, the neutrino can be considered stopped. Normal estimations of the free streaming lower limit mass are of the order of $10^{12} M_\odot$ too, although current ideas about the nature of dark matter favour CDM.

Once we have considered the question of when an inhomogeneity is unstable, and therefore when an overdensity region grows and δ increases, let us briefly speak about the $\delta(t)$ function, or its equivalent, $\delta(R)$. Again a pre-Recombination treatment requires general relativistic tools, Newtonian Mechanics being adequate after Recombination. However, this later epoch is much more complicated from the mathematical point of view, because we know that at present $\delta > 1$, which means that the evolution is non-linear. In the radiation dominated epoch it was $\delta << 1$ and the standard linear perturbation analysis is a very good approximation.

It has been obtained that growth perturbations increase as $\delta \propto t$, therefore $\delta \propto R^2$, during the radiation dominated epoch before entering the Acoustic era. During this Acoustic era, if it really existed, it is apparent that δ is a constant, or, rather, periodic. After Recombination, inhomogeneities grow as $\delta \propto t^{2/3}$, therefore $\delta \propto R$, until δ is closer to unity. Then, the simple linear analysis technique is no longer adequate. Non-linear calculations suggest that first $\delta \propto R^2$, afterwards $\delta \propto R^3$, but then the hierarchical models, as commented below, constitute the most widely accepted technique to study this recent evolution.

Figure 15 plots $\delta(R)$, but is only a rough description due to the many factors which are at present poorly understood.

4.2.2 CDM Hierarchical Models

As mentioned above, current theoretical models of galaxy formation and evolution are based on historical papers, in particular those by Press and Schechter (1974) and by White and Rees (1978). Other pioneering papers (e.g. Gunn, J.E. and Gott, J.R., 1972; Gunn, 1977, and references therein) have also contributed to the presently accepted scenario, which has had a considerable success in explaining a large variety of galaxy and clustering properties.

Previous studies led by Zeldovich (e.g. Zeldovich, 1970; Sunyaev and Zeldovich, 1972) considered that small mass objects formed from nonlinear processes in clusters with a larger hot dark matter mass. Press and Schechter analyzed the opposite point of view, according to which larger mass objects form from the non-linear interaction of smaller masses, with these being formed before. Some of these ideas were suggested by Peebles (1965). Press and Schechter (1974) did not mention dark matter, but a "gas" of self-gravitating mass.

Starting from an initial spectrum of perturbations shortly after Recombination, baryonic aggregates with a small mass condensed, merged to form larger condensations, which in turn merged and so on. In this way, the condensation proceeded to larger and larger scales, at later and later times. They proposed that this merging series very weakly depended on the spectrum of seed masses initially assumed. "When the condensation has proceeded to scales much larger than the seed scale, the *gas* should have essentially no memory of its initial scale and the condensation process should approach a self-similar solution".

One of the most decisive papers in the modern history of Astronomy was written by White and Rees (1978), which may be considered the progenitor of nearly all current theoretical models of galaxy formation. This model adopted the hierarchical clustering scenario proposed by Press and Schechter (1974) but introduced two new basic ingredients: dark matter and the cooling of the baryon system to produce the visible component of galaxies.

First, these authors proposed $\Omega \sim 0.2$, which is certainly close to present-day estimates. They considered that the baryon to dark matter fraction in the Universe should not very much differ from that in a rich cluster, like Coma, from where they adopted M/L =400. This is in agreement with modern estimates of $150h \leq M/L_B \leq 500h$ (Bahcall, Lubin and Dorman, 1995). As L_B, the blue radiation energy density is of the order of $2 \times 10^8 hL_\odot Mpc^{-3}$ (see for instance, Zucca et al., 1997, for a current value), $\Omega \sim 0.2$ is then deduced.

From this, they proposed that approximately 80% of this matter was dark matter and of the remaining 20%, half was still uncondensed baryons and the other half constituted the luminous component.

In this scenario, small halos formed first through a merging process; the first generation of halos produced a new generation and so on. New generations in the hierarchy are therefore born later and are more massive. The process is interrupted by the finite time of the Universe, and therefore no clustering is to be expected at a large enough scale. The smaller scale virialized systems merged into an amorphous whole, mainly constituted of dark matter but also of gas, as a minor component. When the gas cooled it fell into the centre of the DM halo and there became sufficiently concentrated to produce stellar collapses, which rendered it visible.

"When a halo is disrupted in a larger system the luminous galaxy in its core can preserve its identity because dissipation has made it more concentrated than the surrounding dark material" wrote White and Rees. Therefore small galaxies could be reminiscent of the first generation halos. As they formed earlier, dwarf galaxies could have a small mass and a higher density. When these small halos with a luminous core merged to produce a larger halo with a larger luminous core, the small baryonic concentration would not be destroyed and should be identifiable as orbiting the large galaxy. The familiar observation of a large galaxy, such as ours, surrounded by many dwarf galaxies would then be explained in a very natural way.

Note that this elegant idea could be in conflict with current interpretations of the observations, which seem to indicate that dwarf galaxies have their own halo and they are not so "dwarf" as they may possess specially massive dark halos. We will return to this point when dealing with the magnetic hypothesis.

In more detail, the fate of the gas would depend on other factors. When two or more smaller halos merge, there is an intense heating, produced by shocks during the violent relaxation that accompanies the formation of the halo. The gas could be heated until it reaches a pressure-supported state. At a temperature of about 10^4 K it would be ionized and able to cool radiatively, via bremsstrahlung, recombination, and so on. The cooling process would settle the concentrated gas into the centre and produce stars, which become a visible component. The pressure-supported gas contraction would be quasi-static. But this slow concentration could be abruptly truncated by a new merging. Therefore, a visible baryonic component would be formed only when the radiative cooling time is less than the typical dynamic or merging time.

Therefore "the luminous material that condensed in their centers may nevertheless have survived to the present day in identifiable stellar systems" (White and Rees, 1978). For instance, this would be not only the case of satellite dwarf galaxies but also of large galaxies within a rich cluster like Coma. Such large clusters would possess a very large common halo, rather than small individual halos. By merging, because of the violent relaxation, halos virialize very fast and lose any internal structure other than the baryonic cores.

4.2.3 Recent developments

After these important papers in which the basic scenario was outlined and the general assumptions justified, it was necessary to develop more detailed models, mainly numerical and N-body simulations due to the high complexity of the various physical processes involved.

A schematic list of models and reviews is now given, which should be completed with references therein: Cole (1991), White and Frenk (1991), Navarro and White (1993), Kauffmann, Guidernoni and White (1994), Cole et al. (1994), Navarro, Frenk and White (1996, 1997), Lacey and Cole (1993), Avila-Reese, Firmani and Hernandez (1998), Avila-Reese et al. (1999), Sensui, Funato and Makino (1999), Salvador-Sole, Solanes and Manrique (1998), Baugh et al. (1999), Subramanian, Cen and Ostriker (1999), Steinmetz (1999), van den Bosch (1999) and a large series of papers, reflecting the importance of the topic.

Models can be classified as "semi-analytical" (in which some processes are given a simplified treatment assuming simple recipes, based on either previous numerical calculation or on theoretical ideas), numerical simulations (e.g. hydrodynamical simulations, collision-less simulations) N-body simulations (the most widely used) and even analytical. Some hybrid models are difficult to classify in this scheme.

It is first necessary to adopt a cosmological model, the most popular one being the "standard" CDM (with $\Omega = 1$, $h = 0.5$, for instance) or the ΛCDM (more in consonance with current values, $\Omega = 0.3$, $\Lambda = 0.7$, h= 0.65). A primordial fluctuation spectrum must often be adopted, usually a power law $P(k) \propto k^n$, with n ranging from 0 to -1.5 (for example), where k is the wave number. Another important parameter used by most models is σ_8. In general, the variance σ is defined as $< \delta^2 >^{1/2}$; then σ_8 is the present value for a scale-length of 8 Mpc. This parameter is adopted "a priori" taking into account the present large-scale structure, rather than considering a real free parameter. Usual values adopted are $\sigma_8 \sim 0.6$ for the standard CDM and $\sigma_8 \sim 1$ for the ΛCDM.

Other parameters characterize the calculation methods. For instance the initial redshift, the number of particles in N-body simulations and the box in Mpc^3 in which the calculations are performed. The so called "Virgo consortium" (Jenkins et al. 1997) is able to handle 256^3 particles and a large volume of the order of 60 Mpc. Parameters controlling the resolution of the simulation and the efficiency with which gas cools have a higher influence on the results (Kay et al. 1999).

These models not only deal with the formation of halos, but also with the ability of gas to form stars, with matter and energy outputs, mainly due to supernova explosions, the evolution of the baryonic component, the explanation of the Hubble Sequence, how spirals merge to produce spirals and so on. From our point of view, the rotation of galaxies strongly depends on the structure of the halos, which is determined in the first stage of the computations. The latest evolution of visible galaxies is, paradoxically, the most difficult to understand and to model. For instance, the Initial Mass Function (IMF) is largely unknown and yet is decisive in galactic evolution.

The hierarchical process of merging, the formation and internal structure of dark matter halos is said to be the best known process. This could be due, in part, to the relative simplicity of the process, but also to the evident fact that it is easier to make predictions about the unobservable. In general, even if some observable facts remain insufficiently explained, these families of theoretical models provide a very satisfactory basis to interpret any evolutionary and morphological problem.

4.2.4 General remarks

The basic scenario cannot be accepted without discussion. In the CDM model, in which small structures form first, it is predicted that at a large enough scale no structure should be encountered and that the density distribution should be completely random. This random distribution should be found at scales larger than about 30 Mpc. However, this might not be the case, as there is a large body of evidence suggesting a regular large structure forming a lattice at a larger scale. Broadhurst et al. (1990) found 10 periodic peaks separated by about 128 h^{-1}Mpc in a pencil beam survey, which cannot be due to chance. Einasto et al. (1994) presented very clear evidence of a regular network with superclusters residing in chains separated by voids of diameters $100h^{-1}$Mpc. Such a regular lattice has also been confirmed by other authors (Tucker et al. 1997, Landy et al. 1996, Einasto et al. 1997 and others). Tully et al. (1992) compared the structure with a three dimensional chess-board. Similar regularity has been found in the distribution of QSO absorption-line systems (Quashnock et al. 1996) and in the CMB spectrum (Atrio-Barandela et al. 1997). The Tartu Group has been specially active in demonstrating this large-scale structure (Toomet et al. 1999).

Typical sizes of the lattice elements would be about 100-150 h^{-1}Mpc, but the regularity in the alignment of these elements can be detected for much greater distances. In the Tully et al. (1992) supercluster distribution a straight line consisting of a chain of superclusters can be identified, from Tucana to Ursa Major, or even to Draco, in other words, a straight-line chain 700 h^{-1}Mpc long. We will return to this point when discussing the cosmological magnetic field.

These observations have been rejected by many authors. The regularity found by Broadhurst et al. (1990) has not been found in other directions, but if a lattice is formed by filaments and voids, that is precisely what should be expected. Only in particular selected beams would a periodicity be detected. In the power spectrum of the Point Source Catalogue redshift survey (Sutherland et al. 1999) no periodicities, spikes or preferred directions were found. There was only the marginal evidence of a "step" in the power spectrum at $k \sim 0.08hMpc^{-1}$, but this was just a 2σ effect that the authors considered a statistical fluctuation.

The possible crystal large scale structure is therefore under debate at present. Correlation function analysis is probably not appropriate to study a lattice of filaments and sheets, which would be somewhat deformable, elastic-like, due, for instance, to the gravity action caused by the largest superclusters. Other statistical methods to detect "foam lattices" should be developed. The evidence of a crystal-like structure for scales larger than about 100 Mpc is overwhelming.

Even if this possible observational fact were in complete contradiction with the hierarchical CDM models, strictly speaking, in practice, it would not invalidate them. It could be that, within a large structure ($\sim$ 100 Mpc), the models would be available at a much shorter scale ($\leq$ 30 Mpc). Another explanation for the large scale should be sought, but the smaller scale models could remain valid. Current theoretical models, instead of an initial random distribution of δ, would start with a very large wide filament-sheet lattice as the initial condition. In practice, the existence of a large scale crystal is not incompatible with CDM models.

In addition, the hypothesis of a fractal universe without upper limits (Sylos-Labini, Monturi and Pietronero 1998) should be borne in mind.

Two quasi-philosophical criticisms can always be made of numerical models. If they assume a hypothesis and adjust a number of free parameters to agree with observations, the conclusion is that if the hypothesis is true, the set of free parameters proposed is correct, but the hypothesis has not been proved to be true. In the best case, the hypothesis is just compatible with observations.

Furthermore, for N-body simulations, if we find results matching observational facts, we know that the physics used is able to explain these facts, but we are still unaware of the in-depth explanation.

4.2.5 Some successes and failures

Following Frenk et al. (1997) some basic facts such as the abundance of DM halos, their merging history and their internal structure are reasonably well understood. Semi-analytical models satisfactorily explain the luminosity function, the number counts and colours, the evolution of the Hubble sequence, the morphological type, the history of star formation, etc. However, no model has succeeded in producing a faint end slope of the galaxy luminosity function flatter than $\alpha \approx 1.5$, whilst the observations indicate $\alpha \approx 1$.

Reasonable results are also obtained for the morphologies of galaxies. Galaxies with disks and bulges are directly obtained. They may merge producing an elliptical galaxy. There is increasing evidence that ellipticals arise from the mergings of spirals. Apart from the theoretical models, Pfenninger (1997) vigorously argues in favour of the evolution from the so called "late" to the so called "early" galactic types. There is general agreement on this, though an elliptical may then develop a new disk. However, the gaseous disks obtained are too small, due to the loss of angular momentum to the halo when they form. Note that in the early interesting models by Larson (1974) where no dark matter was considered, just gas and stars, and the calculations were not obtained from a cosmological scenario, no combination of free parameters was able to produce large enough disks. After many years the formation of disks is still a poorly known process.

One of the most remarkable successes is the ability of semi-analytical models to match the counts of faint galaxies as a function of magnitude, redshift and morphology, particularly when standard CDM cosmology is adopted. Frenk et al. (1997) wrote "This agreement is the most striking indication so far that the models contain some element of truth". The redshift distribution of galaxies with magnitudes in the range 22.5<B<24.0 is reproduced in Fig. 16. This agreement was only obtained when the data from Cowie et al. (1996) and the Miller-Scalo IMF become available.

The Lyman Break galaxies (Steidel et al. 1996) probably constitute the first generation of star forming galaxies. They are therefore a challenge for theoretical models, especially considering that their adjustable parameters are set taking into account z=0 properties. Lyman Break galaxies have little emission in the U filter, because the redshifted Lyman Break with rest-wavelength at 912Ålies redwards from the wavelength range of the U filter, while redder filters are still unaffected and transmit a higher signal. This is what should be expected for galaxies with $z \geq 3$ and what Keck has spectroscopically confirmed. This discovery was disconcerting for hierarchical CDM models as they had predicted that galaxy formation was a very recent phenomenon (with z≈1). It seems, however, that the use of the Miller-Scalo IMF instead of the earlier Scalo IMF again provides an agreement with the observations of the Lyman Break galaxies. Furthermore, a readjustment of the σ_8 parameter is required. Agreement is obtained for both the standard CDM and the ΛCDM models, both of which predict a similar early star formation history. Some of the present galaxies could have had a Lyman Break progenitor, specially the brightest galaxies. A bright spiral galaxy could be the descendant of a single fairly massive Lyman Break object; a bright elliptical could be the descendant of two less massive ones, merging at more recent epochs (Baugh et al., 1998; Frenk et al. 1997; Gobernato et al. 1998). As studied by Baugh et al. (1999), a remarkable success of the theory is that models can be adjusted so that the agreement with the z=0 galaxy clustering (Postman et al 1998 and others) also agrees with the clustering data at z=3 given by Adelberger et al. (1998). Figure 17 plots the theoretically obtained present B luminosity.

Salvador Solé et al. (1998) took into account that continuous accretion between mergers and tiny mass captures have a very different effect from notable mergers. Between abrupt major mergers, the central parts of halos grow steadily and their virial radius continues to expand.

4.2.6 CDM Models, halo structure and rotation of spirals

CDM models predict a halo structure which is responsible for the rotation curve of the spiral galaxies. Halo structures and rotation curves are therefore closely connected problems. Let us assume that halos are spherical.

Gunn and Gott (1972) concluded that the gravitational collapse could lead to the formation of virialized halos with almost isothermal profiles. A tempting assumption for the halo density distribution is therefore the so called non-singular "isothermal" sphere

$$\rho(R) = \frac{\rho_0}{1 + \left(\frac{R}{R_c}\right)^2} \tag{71}$$

defined with two parameters: the central density, ρ_0, and the core radius, R_c. In this isothermal profile $(d\rho/dR)(R = 0) = 0$ and $\rho(R = 0) = \rho_o$ is finite, two desirable properties for a density profile. Out to $R \sim R_c$, the density remains more or less flat, i.e., there is a "core" of radius R_c. Note that, in this case, the "circular velocity", V, is defined as

$$V^2(R) = \frac{GM(R)}{R} \tag{72}$$

where $M(R)$ is the mass in a sphere of radius R. It can then be deduced that the circular velocity is given by

$$V^2(R) = 4\pi G \rho_0 R_c^2 \left[1 - \frac{R}{R_c} \arctan \frac{R}{R_c}\right] \tag{73}$$

which is an increasing function of R, asymptotically reaching $V_{max} = V(R = \infty)$ given by

$$V_{max} = \sqrt{4\pi G\rho_0 R_c^2} \tag{74}$$

which might be an undesirable property (an asymptotically Keplerian curve would be preferable).

Other types of halos have been reviewed by Bertschinger (1998) and others. Recently Navarro, Frenk and White (1996, 1997) deduced from their CDM models that halos should be described by the so called "universal" or NFW profiles

$$\frac{\rho(R)}{\rho_{crit}} = \frac{\delta_c}{\frac{R}{R_s}\left(1 + \frac{R}{R_s}\right)^2} \tag{75}$$

where ρ_{crit} is the density of the critical Einstein-de Sitter Universe

$$\rho_{crit} = \frac{3H^2}{8\pi G} \tag{76}$$

and δ_c, the characteristic contrast density (dimensionless) and R_s, the scale radius, are the parameters of the profile. It is singular, $\rho(R = 0) = \infty$, which is certainly an "unpleasant" property, even if the mass $M(0)$ converges. The NFW profile was called "universal" because the authors found it in a large variety of halo masses, spanning 4 orders of magnitude, from individual galaxies to cluster halos, as well as for a large variety of cosmological scenarios. Some authors (e.g. Avila-Reese, Firmani and Hernandez, 1998) deduced that the density profile depends on the environment, with the NFW appropriate only for isolated halos. The circular velocity of this NFW halo can be calculated by

$$\left(\frac{V_c(r)}{V_{200}}\right)^2 = \frac{1}{x}\frac{\ln(1+cx) - \frac{cx}{1+cx}}{\ln(1+c) - \frac{c}{1+c}} \tag{77}$$

where V_{200} is the circular velocity at R_{200}, called the "Virial" radius. This virial radius is that radius for which $<\rho>= 200\rho_{crit}$, where $<\rho>$ is the mean density in a sphere of radius R_{200}. Cole and Lacey (1996) showed that this radius approximately separates the virialized and infall regions. The parameter c, called the concentration, is defined as

$$c = \frac{R_{200}}{R_s} \tag{78}$$

and is dimensionless. x is simply R/R_{200}.

As $V_{200}^2 = GM_{200}/R_{200} = G(200{\times}3H^2/8\pi G)(4\pi(R_{200})^3/3)/R_{200} = 100H_0^2R_{200}^2$, we have $V_{200} = 10H_0R_{200}$, or

$$V_{200} = R_{200}h \tag{79}$$

if we measure V_{200} in kms^{-1} and R_{200} in kpc. Also:

$$M_{200} = (200\rho_{crit})\frac{4}{3}\pi R_{200}^3 = 100\frac{H^2}{G}R_{200}^3 \tag{80}$$

therefore $M_{200} \propto R_{200}^3 \propto V_{200}^3$. If the luminous mass were proportional to the halo mass, M_{200}, and if V_{opt} were related to V_{200} (V_{opt} is the disk velocity at R_{opt}, the optical radius), then a relation similar to the observational Tully-Fisher relation would be obtained. In conventional astronomical units

$$M_{200} = 2.33 \times 10^5 V_{200}^3 M_\odot \tag{81}$$

where V_{200} is to be expressed in km/s. The Tully-Fisher relation clearly establishes a relation between the luminosity and some power of the optical rotational velocity, $L \propto V_{opt}^3$ or $L \propto V_{opt}^4$. (The exponent depends on the wavelength of the observations. The higher value of 4 is for the infrared. See van der Bosch (1999) for a recent critical review).

The obtention of the Tully-Fisher relation from the outcoming halo density distribution presents some problems. The slope obtained and the scattering of the points agree with the observations, but the theoretical curve is displaced with the observational curve (Frenk et al. 1997). Or equivalently, it is possible to vary the free parameters to match the Tully-Fisher relation but then the amplitude of the galaxy luminosity function is not matched. This is at present a failure of theoretical models. The number of predicted dark halos is excessive. Navarro, Frenk and White (1996) suggested several possibilities: other cosmological parameters, the existence of a large number of halos with no visible component, the non-detection of many existent low surface brightness galaxies, etc.

Even if the observational Tully-Fisher relation is basically reproduced, there is still no convincing explanation for the numerical outputs. As Navarro (1998) remarks: "Our analysis has made use of this surprisingly tight relation between disk luminosity and rotation speed but provides no firm clues to elucidate its origin".

There is a general argument based on Shu (1982) that does not explain the Tully-Fisher relation but does introduce some light. If the total luminosity of a galaxy is roughly $L \propto \Sigma_0 R_{opt}^2$ (where Σ_0 is the central surface brightness, notably the same for all spirals, and R_{opt} the optical radius) and if the total mass within the optical radius is roughly $\mathcal{M} \propto R_{opt}V_{opt}^2$ (where V_{opt} is the asymptotic observed velocity) and if $\mathcal{M}/L$ is roughly a constant, then $R_{opt} \propto V_{opt}^2/\Sigma_0$. If Σ_0 is really a constant $R_{opt} \propto V_{opt}^2$ and therefore $L \propto V_{opt}^4$. This argument relies on the constancy of Σ_0, which so far has no explanation.

The NFW circular velocity reaches a maximum at $R \approx 2R_s = 2R_{200}/c$ and declines beyond that radius. NFW density profiles are two-parametric and it is possible to choose V_{200} and c to characterize the halos, or the equivalent set of characteristic density and halo mass. A very exciting result of these theoretical models is that the two free parameters show a clear correlation. The reason behind this is that the halo density reflects, and is proportional to, the true density when the halo was formed, with the initial small halos being denser because they formed earlier, when the density of the expanding Universe was higher. But we also know that, due to the hierarchical halo formation, more massive halos were born later. Then, the existence of decreasing functions $\rho_c(t)$ and $M_{200}(t)$ implies a correlation between $\rho_c(t)$ and $M_{200}(t)$. Therefore, in practice, rotation curves are intrinsically one-parametric. Figure 18 plots the circular velocity curves for different values of the concentration, c. Low values of the concentration parameters denote slowly rising curves, i.e. small ancient galaxies.

We saw above that the "maximum disk" hypothesis provides halo circular velocities. Do NFW halo profiles fit these profiles? Are disks really maxima? Do isothermal profiles provide a better fit than NFW profiles? A comparison of the theoretically predicted halos and the observations is necessary to answer these questions and to test the models. This comparison was made by Navarro (1998), who adopted about 100 disk galaxies from published observations and tried to deduce the NFW free parameter in each case. This analysis was also made under the isothermal halo assumption.

For this task, we should not forget that galaxies also have visible components, i.e. that spirals possess a disk and a bulge. For the surface brightness of the disk, Navarro (1998) assumed, as usual, an exponential disk

$$\Sigma(R) = \Sigma_0 e^{-R/R_d} \tag{82}$$

where $\Sigma_0 = \Sigma(R = 0)$ and R_d, the disk radial scale length, are two parameters. Exponential optical disks constitute a reasonable zeroth-order description, but there is perhaps a misuse in the literature, as above discussed. It is however an appropriate and almost a necessary assumption in studies similar to that of Navarro.

For the surface brightness of the bulge it is assumed that

$$\Sigma_b(R) = \Sigma_{b0} e^{-(R/R_b)^4} \tag{83}$$

where Σ_{b0} and R_b are two parameters. Navarro assumed the same M/L ratio in the bulge and in the disk, and taht this ratio was another free parameter.

It is also usually assumed that the halo responds "adiabatically" to the growth of the disk (Barnes and White, 1984; Flores et al. 1993), which was also assumed by Navarro, Frenk and White (1996, 1997). This means a variation of the NFW profile within the disk region. Certainly, the formation of the disk must somewhat modify the halo density profile, probably in a way that is very difficult to model. This assumption was introduced in order to avoid disagreements with observations of the rotation of dwarf galaxies and therefore, as a correction of an initial theoretical failure. In fact, the rotation curve of dwarf galaxies indicates that the halo circular velocity rises almost linearly, which would mean a constant density (i.e. a halo core) in clear contradiction with the NFW profiles. As mentioned above, this hypothesis of the adiabatic response of the halo to the formation of the disk also alleviates the problem of the halo-disk conspiracy.

The results indicate that theoretical models must introduce a higher degree of sophistication, because even isothermal profiles give similar or better results, specially for low surface brightness galaxies. Moreover, observational rotation curves are very often well fitted by halos with no disk and no bulge! This type of fitting is meaningless, but Cosmology could benefit from it. The concentration parameter, c, obtained after this peculiar halo-only fitting provides an upper limit which can be compared with the theoretical predictions on c. The standard CDM model in general predicts higher concentrations than the upper limits obtained. Therefore, if the theoretical models are considered an efficient basis to interpret the observations, then cosmological models with small $\Omega(\sim 0.3)$ and large $\Lambda(\sim 0.7)$ are favoured.

A puzzling observation within the DM interpretation of rotation curves is the absence of correlation between the asymptotic velocity of disks and the orbital velocity in binary systems. If the asymptotic velocity, V_{rot}, is found in a region dynamically dominated by the halo, and the orbital velocity, V_{orb}, of a galaxy considered the secondary would, clearly, reflect the total mass of the primary, V_{rot} and V_{orb} should correlate. Navarro (1998) seeks the explanation in his Fig. 19 (top-right: $\log V_{200}$ versus $\log V_{rot}$). If we observe the solid line in this figure, we see that disks with $V_{rot} \leq 150kms^{-1}$ have $V_{200} > V_{rot}$ and disks with $V_{rot} > 150kms^{-1}$ are all predicted to have similar halo velocities, $V_{200} \sim 200kms^{-1}$. Then, disk-dominated galaxies would be surrounded by halos of approximately the same mass.

This explanation of rotation curves remains incomplete. Samples do not contain dwarf galaxies. The Navarro sample is probably also lacking early bulge-rich galaxies. In Fig. 26 there are two other curves, the M/L =h curve and the varying M/L curve, which are not unreasonable (for instance, $V_{rot} = V_{200}$ implies very small variations of M/L) but which, however, do not imply the same conclusion. Even if the solid line shows us a change in slope, the increasing function does not firmly predict the complete absence of correlation. The discussion of Navarro concerning this point is very illustrative but the puzzling behaviour of binary galaxies is not completely cleared up. The existence of an upper limit of the halo mass needs further justification.

Furthermore, Navarro and Steimetz (1999) find it difficult to reconcile the theory with data for the Milky Way and with the Tully-Fisher relation; they consider that substantial revision of the theoretical models is needed.

As a general conclusion, observational rotation curves are not incompatible with NFW halos, but the confrontation seems somewhat discouraging. We have examined the work of Navarro (1998) in some detail because it is probably the most serious and complete study linking the observed properties and the model outputs for rotation curves. However, either the observations do not constitute a proof of the CDM models, or dynamic ingredients other than halo and disk density profiles are necessary to study the rotation of spirals. Considering the success of the models in explaining and predicting other observational facts, we would suggest this second possibility as more plausible. In particular, we will later argue that ignoring magnetic fields in the interpretation of the rotation curves could be unrealistic.

4.3 MOND

The interpretation of the rotation curve of spiral galaxies is based on the assumption that Newtonian Dynamics is valid. This assumption is not accepted by the so called "Modified Newtonian Dynamics" (MOND) developed and discussed by Milgrom (1983a,b,c), Sanders(1990) and others. An implicit rule in our approach to Cosmology is that our physical laws are valid everywhere, unless they lead us to unacceptable conclusions. This law is therefore apparently violated. But Newtonian Dynamics was established after considering nearby astronomical phenomena, and we are allowed to modify it now, when we are aware of distant large-scale phenomena, unknown in Newton's time. Moreover, MOND aims not only to explain the rotation curve of spirals, but to propose an alternative theory of Gravitation and/or Dynamics. With respect to galaxies, the introduction of MOND provides very remarkable fits, rendering this theory a very interesting alternative.

As an introduction, let us consider the outermost disk, where the galactic mass can be considered a central point producing a gravitational potential that predicts a Keplerian decrease. But instead of the standard form for the acceleration of gravity, $g = GM/r^2$, assume that it is expressed as $K(M)/r$, where $K(M)$ is a constant depending only on the galactic mass. If this force is matched by the centrifugal force, θ^2/r, we readily obtain $\theta^2 = K(M)$, therefore obtaining that θ is a constant and thus the rotation curve paradox is automatically solved, without the need for dark matter, at least for bright galaxies.

What would be the dependence of K on M? A good assumption in the absence of dark matter could be that the M/L ratio is independent of R. Therefore, if $K(M) = xM^{1/2}$ with x being any constant, then we would directly obtain $M \propto \theta^4$, and therefore, $L \propto \theta^4$, which is the Tully-Fisher relation. Thus, we have already solved the two basic problems of the rotation of spirals: flat rotation curves and the Tully-Fisher relation. Therefore, g would depend on $M^{1/2}$, or equivalently, on $(GM)^{1/2}$. Hence, g=constant$(GM)^{1/2}R^{-1}$. The constant would have the dimensions of the square root of an acceleration and would be a universal constant. Let us call this acceleration a_0. Finally, $g = (GMa_0)^{1/2}R^{-1}$. The constant a_0 was introduced by Milgrom (1983a).

Let us estimate its value. If this kind of gravity is supported by rotation, $\theta^4 = GMa_0$. Taking $\theta \sim 200 kms^{-1}$ and $M = 10^{11}M_{\odot}$, as typical values in a spiral (without dark matter), we obtain $a_0 \sim 1.2 \times 10^{-8} cms^{-2}$. More precise estimations provide $2 \times 10^{-8} cms^{-2}$ (Milgrom, 1983b, for $H_0 = 50 kms^{-1}Mpc^{-1}$)

or $1.2 \times 10^{-8} cm s^{-2}$ (Begeman et al. 1991, with $H_0 = 75$).

But we know that $g = GM/R^2$ for familiar dynamical systems. It could be that the standard expression is valid for small R, and the tested expression $(GMa_0)^{1/2}R^{-1}$ is valid for large R. In the whole region g could obey the sum of the two

$$g = GM/R^2 + \frac{(GMa_0)^{1/2}}{R} \tag{84}$$

the first term would predominate at low R, and the second at large R. This expression was first proposed by Sanders (1990).

The transition region would be characterized by a similar order of magnitude of the two terms, i.e. for R(transition) around $\sqrt{GM/a_0}$. For M of the order of a galactic mass, the transition would take place at around 10 kpc (but a galaxy is not a central point mass), thus suggesting that at large radii, the new gravitational term would predominate, which solves both the flat rotation curve and the Tully-Fisher problems.

This equation is just a first simple example to see how our problems can be solved by modifying Newton's Universal Law of Gravitation. There is a second way, consisting in accepting this law, but modifying Newton's Second Law, $\vec{F} = m\vec{a}$. Which procedure is the best? Apparently, the first one is preferable, as the whole of Newtonian Dynamics remains valid and we just modify a law that was proposed by Newton from the observations, but without claiming it to be a fundamental principle. In the same way, Einstein's Field Equations could be modified without rejecting General Relativity. Therefore, modifying the gravitational law would be a much softer procedure than modifying the Second Law. However, for gravitational purposes, both procedures are equivalent.

Let us reconsider the problem from a more general point of view.

In MOND, Milgrom (1983a) proposed, instead of the Second Law, that

$$\vec{F} = m\mu\left(\frac{a}{a_0}\right)\vec{a} \tag{85}$$

with $\mu(x)$ being a function to be determined, of which we only know

$$\mu(x \gg 1) = 1 \tag{86}$$

$$\mu(x \ll 1) = x \tag{87}$$

i.e., for low accelerations, much less than a_0, the Second Law would be substituted by $\vec{F} = m(a/a_0)\vec{a}$, being the force proportional to the squared acceleration. In this way, in a galaxy, with $a = \theta^2/R$ we would have $GM/R^2 = (\theta^4/R^2)a_0$ again solving both the flat rotation problem and the Tully-Fisher relation.

Alternatively, the modification of the gravitational law could be expressed in a general form as

$$\vec{g} = a_0 I^{-1}(g_N/a_0)\vec{e}_N \tag{88}$$

where I is an unknown function, and therefore its inverse, I^{-1}, is also an unknown function; g_N is the standard Newtonian gravitational acceleration and $\vec{e}_N$ is a unit vector with the standard direction of $\vec{g}_N$.

The directions of all vectors are the same, and therefore we can denote our derivations without vector arrows. If g_N is the classical Newton gravitational acceleration we rewrite (85) as

$$\frac{g_N}{a_0} = \mu\left(\frac{a}{a_0}\right)\frac{a}{a_0} \tag{89}$$

The arguments of both functions μ and I^{-1} are any variable, but we will keep the notation $g_N/a_0 = u$ and $a/a_0 = x$, therefore, instead of (89)

$$u = x\mu(x) \tag{90}$$

and, instead of (88)

$$\frac{g}{a_0} = I^{-1}(u) \tag{91}$$

(91) is equivalent to modifying the gravitational force while retaining Newton's Second Law, $g = a$, therefore

$$\frac{g}{a_0} = I^{-1}(u) = \frac{a}{a_0} = x \tag{92}$$

hence

$$u = I(x) \tag{93}$$

With (90)

$$I(x) = x\mu(x) \tag{94}$$

If $x \ll 1$, $\mu(x) = x$, hence $I(x) = x^2$, $I^{-1}(u) = u^{1/2}$.

If $x \gg 1$, $\mu(x) = 1$, hence $I(x) = x$, $I^{-1}(u) = u$.

We have stated that the modification of Newton's Second Law, while retaining the expression of the gravitational force, is equivalent to the modification of the gravitational force and retaining Newton's Second Law. We will show this with two examples, based on the above expressions.

The first example considers the modification of Newton's Second Law, in the way first proposed by Milgrom (1983b), corresponding to

$$\mu(x) = \frac{x}{\sqrt{1+x^2}} \tag{95}$$

This expression is interesting as it is the simplest form of accomplishing our asymptotic conditions (86) and (87). With (95) Newton's Second Law would be replaced by

$$\vec{F} = m\vec{a}\frac{\frac{a}{a_0}}{\sqrt{1+\left(\frac{a}{a_0}\right)^2}} \tag{96}$$

which for $a \gg a_0$, effectively reduces to $\vec{F} = m\vec{a}$.

Then our question becomes what is the equivalent transformation of the gravitational force, i.e. producing the same dynamical effects as the simple proposal of Milgrom (1983b) in (95)? We just need to find I, and then I^{-1} can be inserted in (88). With (94)

$$I(x) = \frac{x^2}{\sqrt{1+x^2}} \tag{97}$$

With (93)

$$u = \frac{x^2}{\sqrt{1+x^2}} \tag{98}$$

hence

$$x = \left(\frac{u^2 + u\sqrt{u^2+4}}{2} \right)^{1/2} \tag{99}$$

The signus (-) provides an non physical solution. With (92)

$$I^{-1}(u) = \left(\frac{u^2 + u\sqrt{u^2+4}}{2} \right)^{1/2} \tag{100}$$

Observe that if $u \gg 1$, then $I^{-1}(u) = u$, which means that in regions where Newton's classical gravitational force is high ($g_N \gg a_0$), the gravitational force coincides with the classical one. But, if $u \ll 1$, $I^{-1}(u) = u^{1/2}$ and $g = (a_0 g_N)^{1/2}$, it is rather different. The complete expression of g would become

$$g = a_0 \left(\frac{(g_N/a_0)^2 + (g_N/a_0)\sqrt{(g_N/a_0)^2 + 4}}{2} \right)^{1/2} \tag{101}$$

an interesting formula because it is obtained without modification of the three general laws of Newtonian Mechanics, explains flat rotation curves without dark matter and explains the Tully-Fisher relation.

As a second example, suppose the inverse problem. We start with a modification of Newtonian gravitational force and want to know the equivalent modification of Newton's Second Law. We must first propose a new form of the gravitational force. We suppose that (86) is the correct expression as we have seen that it also explains flat curves and the Tully-Fisher relation. First, we rewrite (86) as

$$\frac{g}{a_0} = \frac{g_N}{a_0} + \left(\frac{g_N}{a_0} \right)^{1/2} \tag{102}$$

With (88) and $u = g_N/a_0$, this is written

$$I^{-1}(u) = u + u^{1/2} \tag{103}$$

or

$$I(u+u^{1/2})=u \tag{104}$$

With (93)

$$x=u+u^{1/2} \tag{105}$$

or

$$u=\frac{1+2x\pm\sqrt{1+4x}}{2} \tag{106}$$

With (93) again

$$I(x)=\frac{1+2x\pm\sqrt{1+4x}}{2} \tag{107}$$

With (94)

$$\mu(x)=\frac{1+2x\pm\sqrt{1+4x}}{2x} \tag{108}$$

If $x \gg 1$, $\mu(x)=1$, which matches (86). If $x \ll 1$, $\mu(x)=\frac{1\pm1}{2x}$. If we take the signus (+), we have $\mu(x)=x^{-1}$ which is not correct, following (87). We then take the signus (-); then we apparently obtain $\mu(x)=0$. But we should then expand $\sqrt{1+4x}\sim 1+2x-2x^2$ and therefore $\mu(x)=x$ in agreement with (87). The signus (-) therefore gives us the physical solution. The complete modified Second Law would then be

$$\vec{F}=m\vec{a}\left(\frac{1+2\frac{a}{a_0}-\sqrt{1+4\frac{a}{a_0}}}{2\frac{a}{a_0}}\right) \tag{109}$$

which could be a general expression, Newton's Second Law could be just an approximation for high accelerations.

4.3.1 MOND applied to different astrophysical systems

Milgrom (1983a) showed that this theory of gravitation could explain the advance of Mercury's perihelion, first interpreted by Leverier as due to Vulcano, hypothetical intramercurial planet, later "observed" by Lescarbault, and finally explained by General Relativity. Another interesting application concerns the distance of Oort's cometary cloud, as commented below. Milgrom (1983b) applied MOND to the problem of the vertical distribution of stars in relation to the velocity dispersion.

Of course, MOND was successful when applied to galaxies, as it was originally intended to explain rotation curves. It is not remarkable that MOND explains rotation curves, but what is really remarkable is that a very large variety of galaxies can be fitted under this hypothesis, with only one parameter, i.e. the M/L ratio of the bulge.

This task of fitting real data was undertaken by Begeman, Broeils and Sanders (1991) and later continued by Sanders (1996) and Sanders and Verheijen

(1998). The method consists basically of obtaining $\vec{g}_N$ by classical procedures and then considering equation (84) to fit the results. With the sole exception of NGC 2841, the results were very good, excellent in some cases, even better than the multiparameter fitting considering a dark matter halo. About 80 spiral galaxies with a large variety of luminosities and types were compatible with MOND.

The success was particularly interesting in the case of Low Surface Brightness galaxies. These galaxies can be considered to have a low surface density, too, in the absence of dark matter. Accelerations are therefore so low that the whole galaxy can be considered within the MOND regime. Milgrom (1983a,b,c) even deduced that positive slopes in the rotation curve could be expected, which was later confirmed by Casertano and van Gorkom (1991). These galaxies were studied by de Block and McGaugh (1998), who found reasonable and constant M/L ratios, when the use of classical Newtonian Dynamics provides M/L ratios ranging from 10 to 75 (van der Hulst et al., 1993).

As mentioned above, dwarf spheroidal galaxies are also interpreted as being characterized by very large M/L ratios, in the range 10-100 (Mateo, 1994; Vogt et al., 1995). Gerhard (1994) applied MOND to 7 dwarf spheroidals, without finding any agreement. This negative result was confirmed by Gerhard and Spergel (1992), finding unacceptable differences in the M/L ratios required, but Milgrom (1995) reanalyzed these 7 galaxies and obtained a reasonable agreement between MOND and the observations.

De Block and McGaugh (1998) studied 15 galaxies with low surface brightness, finding a low dispersion in the M/L ratios. Sanders and Verheijen (1998) carried out the analysis in the infrared K' band, where extinction and recent star formation effects do not alter the photometric profiles, and obtained values in good agreement with those predicted by MOND.

Rodrigo-Blanco and Pérez-Mercader (1998) have directly considered the modification of the Newtonian acceleration of gravity from the rotation curve of 9 galaxies, also without the need of dark matter.

Van den Bosch and Dalcanton (2000) compared the results obtained with semi-analytical models and with MOND. This search was undertaken because in their opinion "the dark matter scenario is certainly starting to lose its appealing character" due to the mix of baryons, CDM and HDM needed, as well as a non-zero cosmological constant; therefore, other alternatives should be seriously reconsidered. These authors found that both theories can explain rotation curves almost equally well, even if MOND needs a similar amount of fine-tuning.

Milgrom (1983c) also studied a large variety of systems with dark matter problems, such as binary galaxies, small clusters, rich clusters and in particular, Virgo. All these systems are characterized by low acceleration, and no great quantities of dark matter were required. The and White (1988) considered optical and x-ray observations to check MOND in Coma, finding that models without dark-matter are compatible. Sanders (1999) finds a non-negligible difference between the dynamic and the luminous mass, this ratio being about 2, less than that obtained by standard Newtonian Dynamics but far from unity. He attributes this discrepancy to the fact that accelerations in the innermost part of clusters are not much lower than a_0 ($\approx 0.5a_0$). If this application is correct, it could be due to the non-detection of considerable luminous matter at the centre of rich clusters.

Milgrom (1983a) also discussed the determination of mass in clusters by the gravitational lensing method. Qin, Wu and Zou (1995) concluded that no dark matter was needed under the MOND interpretation. The Faber-Jackson (1976) relation, $L \propto \sigma^4$, where σ is the velocity dispersion in an elliptical galaxy, is also explained under Milgrom's hypothesis.

More recently, Milgrom (1997) has studied the filaments that characterize the large-scale structure of the Universe, comparing the M/L ratio obtained with that of Eisenstein, Loeb and Turner (1997) who found $M/L \sim 450h$ in solar units in a filament in the Perseus-Piscis supercluster, in contrast with that of Milgrom of only $M/L \sim 19$, again requiring little or no dark matter.

a_0, with a value of about $2 \times 10^{-8} cms^{-2}$, is very close to that of $H_0 c$ of the order of $6.5 \times 10^{-8} cms^{-2}$, which suggests that MOND could have some implications in Cosmology. Milgrom (1983a,b) suggested that a_0 could be connected with the cosmological constant. In any case, the matter contained in the Universe could be considerably less. Felten (1984) developed a MOND-Cosmology, proposing that the homogeneous and isotropic universe would not be possible for small scales. Sanders (1998) continued the discussion, finding another law for the growth of the cosmological scale factor. This cosmology also provides a suggestive scenario for the development of large scale structures.

4.3.2 Final comments about MOND

A modification of the physical laws should be attempted when all classical hypotheses lead the wrong way. However, such modifications are not uncommon in Physics. In particular, in Astrophysics, we could remember the so called Steady-State Cosmology, which added a term of matter creation. MOND is particularly attractive because, with a single adjustable parameter, and a very limited range of allowable values, it is able to explain the basic facts of galactic dynamics very satisfactorily.

Equations (96) and (109) are not as simple as Newton's equation. Simplicity, beauty and symmetry are apparently non-scientific aesthetic concepts, but have often inspired scientific discoveries and should not be completely absent when a renewal of the fundamental laws in Physics is proposed. Indeed, simplicity is an ingredient in MOND. Equation (95), for example, was proposed by Milgrom because of its simplicity. Nevertheless, MOND equations could be not simple but nevertheless true.

However, Newtonian Dynamics, irrespective of its historic origin, is at present an approximation deduced from General Relativity and therefore it enjoys the protection afforded by this wholly accepted theory. So, proposing corrections to Newtonian Dynamics means rejecting General Relativity, one of the most perfect physical theories. Unless MOND acquires a similar justification by General Relativity, it would remain difficult to be accepted, and to date no such derivation has been reported (Sanders, 1998). For such a task, the modification of Einstein's Field Equations would be more acceptable than a reformulation of the full theory of Relativity.

4.4 The magnetic hypothesis

Following this hypothesis (Nelson, 1988; Battaner et al. 1992, Battaner and Florido, 1995; Battaner, Lesch and Florido, 1999; see also Binney, 1992) the rotation curve of spiral galaxies may be explained by the action of magnetic fields in the disk. If this hypothesis is correct, the cosmological implications would be very important.

In this review about the magnetic scenario we will take into account the following questions:

- Are magnetic fields ignorable in the dynamics of the outer disk?
- What kind of magnetic fields do explain the flat rotation curve?
- What mechanisms may produce these magnetic fields?
- What is the ultimate origin of cosmic magnetic fields?
- What is the overall picture of magnetic fields in cosmology?

4.4.1 Are magnetic fields ignorable?

At large radii, gravity decreases as R^{-2}. In contrast, magnetic fields evolve locally due to gas motions. As in the case of the Sun, at large enough radii, magnetic fields may become more important than gravity, or even dominant.

It is at present evident that $10\mu G$ fields exist in the inner disks. It is increasingly evident that $1\mu G$ fields exist in the intergalactic medium, this point being addressed later. It is therefore to be expected that in the region in between -the outer disk- magnetic fields larger than $1\mu G$ exist, even if they have not been observed.

Then, at some radii the magnetic energy density should reach the order of magnitude of the rotation energy density

$$\frac{1}{2}\rho\theta^2 \sim \frac{B^2}{8\pi} \tag{110}$$

or, equivalently, the Alfven and the rotation speeds should have the same order of magnitude.

This equality of both energy densities will take place for a magnetic field strength that depends on the gas density and on the rotation velocity. Rotation velocities typically range from 50 to 200 kms^{-1}. Typical values of n, the number density of atoms, can be estimated as in our galaxy. Burton (1976) showed a plot in which $n \approx 0.2 atoms cm^{-3}$ at 10 kpc and $n \approx 0.01 atoms cm^{-3}$ at 20 kpc. With an exponential decrease n should be about $2 \times 10^{-3} atoms cm^{-3}$ at 25 kpc. Burton (1992) gives similar values reaching $5 \times 10^{-4} at cm^{-3}$ at about 30 kpc. For other galaxies it should be kept in mind that we measure the surface density in atoms cm^{-2} and to obtain n, the flaring of the layer must be taken into account. For instance, for the Milky Way, the FWHM thickness is about 300 pc at the Sun distance and at $R \sim$20 kpc, it is higher than 1 kpc (Burton, 1992). HI disks appear to have a cut-off at about $10^{19} atoms cm^{-2}$ (Haynes and Broeils, 1997). Van Gorkom (1992) found this cut-off at $4 \times 10^{18} atoms cm^{-2}$. For a thickness of about 1 kpc, this corresponds to $n = 10^{-3} atoms cm^{-3}$. This cut-off is probably due to the ionization of the intergalactic UV radiation, not to a cut-off in the hydrogen itself. However, in our Galaxy, we observe number densities lower than this. We should take $n = 0.3 - 3 \times 10^{-4} atoms cm^{-3}$ in the region of more-or-less flat rotation curves.

The following table gives the magnetic field strength required to produce the same kinetic and magnetic energy densities.

Magnetic fields ~ 0.1 this strength should already have a measurable influence; a magnetic field of this strength would even be a dominant effect. To interpret this table it is therefore to be emphasized that with the strengths given, even gravitation would be negligible.

θ	200	100	50
n			
3×10^{-1}	35	18	8
3×10^{-2}	11	6	3
3×10^{-3}	3	1	0.7
3×10^{-4}	1	0.6	0.3

n in atomscm^{-3}, θ in kms^{-1}, B in μG.

For instance, in our galaxy, the magnetic field would be negligible at the Sun distance, important at 20 kpc and dominant at the rim.

Moreover, we can compare the gravitational attraction and the magnetic force. For an order of magnitude calculation let us adopt the point mass model, $\rho GM/R^2$, where M is the galactic mass (without dark matter) and the magnetic force is, as we will see later, of the order of $(B^2/R)(1/8\pi)$. For a galaxy like the Milky Way we obtain similar values as before. A non-negligible magnetic field should be of the order of $6\mu G$ at R = 10 kpc, of $1\mu G$ at R= 20 kpc and of $0.4\mu G$ at R= 0.4 kpc.

For dwarf late-type galaxies, which are usually considered to need higher dark matter ratios, the magnetic fields required are higher but nevertheless worryingly large. Using the same estimation formula $\rho GM/R^2 \sim B^2/(8\pi R)$ we have approximately

$$B^2 \approx 10^{-8}\frac{\Gamma\Sigma}{RH} \tag{111}$$

where Γ is the visible M/L ratio in solar units, Σ the typical surface brightness in $L_\odot pc^{-2}$, R the radius in kpc and H the scale height in kpc. For typical values, $\Gamma = 1$, $\Sigma = 0.3$, $R = 5$, $H = 0.5$ (from Swaters, 1999) we obtain that a strength of $B \sim 3.5 \times 10^{-5}G$ would produce a force as important as gravitation in the whole galaxy and that $\sim$1/10 this value $\sim 4 \times 10^{-6}$G would be non-negligible. The orders of magnitude obtained by the equality of kinetic and magnetic energy are again similar.

Following the analysis of Vallée (1994), the hypothesis of magnetic-driven rotation curves is unsustainable. The magnetic field strengths required in the model by Battaner et al. (1992) were too high, by at least a factor of 2, as compared to the weaker magnetic field strengths observed. Battaner et al. (1992) indeed required high magnetic fields, of about 6 μG at the rim. However, after the publication of the study by Vallée (1994), Battaner and Florido (1995) recalculated the strength required, by means of a two-dimension model including escape and flaring, obtaining much lower values, of the order of 1μG. These values are not incompatible with observations, for instance reviewed by Vallée (1997), in his exhaustive analysis of cosmic magnetic fields at all scales, and in particular in spiral galaxies. In this review, he collates a number of measurements obtained by other authors and by himself. On the other hand, there are not many measurements available for the outermost region of the disk, as discussed later.

The figures in the above tables are worrying. It can be concluded that *interpreting rotation curves, while ignoring the influence of magnetic fields may be completely unrealistic*. It is therefore remarkable that a fact that may be so far-reaching concerning our cosmological beliefs has been object to such scarce attention.

The magnetic hypothesis takes this fact into consideration and tries to determine whether magnetic fields alone, without requiring any dark matter, and without modifying our physical laws, are able to explain the observed flat and fast rotation curves. The existence of dark matter cannot be completely excluded, but here we explore the extreme case with no DM at all.

Extragalactic magnetic fields Observations are probably still too scarce to reveal the magnitude and distribution of extragalactic magnetic fields. Kronberg (1994) has extensively reviewed all available observations based on synchrotron radiation and its Faraday rotation, and has proposed several properties of which the following are outstanding:

a) Typical values of intergalactic magnetic field strengths are in the range

1-3 μG. These are larger than previously thought, so their influence on a large variety of phenomena must be revised.

b) These values are found nearly everywhere and are noticeably independent of the density of the zone observed. They are found in cluster cores, in clusters and in regions between clusters (e.g. between Coma and A1367, Kim et al. 1989). The first measure of magnetic fields in the intracluster medium was reported by Vallée et al. (1987), finding $\sim 2\mu$G in A 2319. Feretti et al. (1999) have obtained a field strength of between 5 and 10 μG in Abell 119. In superclusters, high value strengths have been reported (e.g. Vallée, 1990, finding about 2 μG in the Virgo Supercluster for the ordered component of the field).

Kronberg has speculated about a ubiquitous magnetic field. In some particular objects, such as radiosources, magnetic fields can be much higher, but this $1\mu G$ background field seems to be ubiquitous. The value of $3\mu G$ is particularly interesting since then the magnetic energy density equals that of CMB, thus suggesting an equipartition of both energies. Note that both energy densities decrease as R^{-4} (being R the cosmic scale factor), but this equipartition, if it exists at all, cannot be primordial, as argued below. However, magnetic fields have never been reported in the large-scale ~100 Mpc sized voids. Only Vallée (1991) has searched for an excess rotation measure in the Bootes Void, estimating that the magnetic field strength was less than 0.1μG.

c) Magnetic fields of this magnitude were also present in quasar absorption line clouds, usually interpreted as pregalactic systems. Therefore pregalactic clouds were magnetized as much as present galaxies. Field strengths of this order have also been measured at redshifts 0.395 and 0.461 (Kronberg, Perry and Zukowski, 1992; Perley and Taylor, 1991).

We favour another global picture that is fully compatible with observations but slightly different to Kronberg's view of ubiquitous 1μG field strength. This global picture is also based on our own theoretical work, which will be commented later. We assume that magnetic fields vanish, or have very small strengths in the large-scale voids, in agreement with Vallée (1991), i.e. in most of the volume of the Universe, but are much higher in the filaments of matter (~100 Mpc long, ~10 Mpc thick) characterizing the large scale structure. Therefore, magnetic fields would be neither ubiquitous nor in energy equipartition with the CMB, but, in any case, they are high, about 1-3μG, in the medium surrounding nearly all galaxies. The medium around galaxies should have 1 μG strengths because this is the value at the particular sites where galaxies lie.

More recently, the review by Eilek (1999) confirms the existence of μG field strength in clusters (even in cluster halos), being much higher at the centre ($B_{\parallel} \sim 50\mu$G in M87, for instance).

Magnetic fields in the outermost region of galactic disks Measurements carried out in this zone have not been reported. By roughly interpolating between the large 10μG fields in the inner disk and the lower than 1μG fields outside the galaxy, we cannot exclude fields $\geq 1\mu G$ in the outermost disk.

Objections to the existence of $1\mu G$ fields at large radii could be raised, with the argument that no detectable synchrotron emission has been reported. However, the non-detection of synchrotron emission cannot be interpreted as the absence of magnetic fields. Kronberg (1995) wrote that "synchrotron radiation can tell us only that magnetic field is present, but not measure its strength".

Despite this pessimistic point of view, let us make some simple estimations. When the relativistic electrons responsible for the synchrotron emission have an energy distribution given by $NdE = N_0 E^{-\gamma} dE$, with N_0 and γ being constants, then the synchrotron intensity can be calculated with (e.g. Pacholczyk, 1970; Ruzmaikin, Shukurov and Sokoloff, 1988)

$$I \propto N_0 \nu^{\frac{1-\gamma}{2}} B_{\perp}^{\frac{1+\gamma}{2}} \tag{112}$$

where $B_{\perp}$ is the component of $\vec{B}$ perpendicular to the line-of-sight. The calculation of $B_{\perp}$ once I is measured, is difficult because N_o is unknown. The spectrum of the synchrotron continuum itself $[I, \nu]$ permits the easy obtention of γ, but not of N_0, meaning the number density of relativistic electrons is unknown. To surmount this difficulty the most usual assumption is that of equipartition.

Equipartition is equivalent to the assumption of equal values of the turbulent and magnetic energy densities and that the energy density is the minimum for a given magnetic field, in which case (Ruzmaikin, Shukurov and Sokoloff, 1988)

$$B^{7/2} \propto \frac{\mathcal{L}}{V} \propto q \propto I \tag{113}$$

where $\mathcal{L}$ is the luminosity of an emitting cloud, V the volume and q the flux.

We will later show that magnetic fields with a gradient slightly less than $B \propto R^{-1}$ can produce a flat rotation curve. If for an estimation we take $B \propto R^{-1}$, then

$$I \propto R^{-7/2} \tag{114}$$

i.e. I decreases much more rapidly than B does (Lisenfeld, 2000). Therefore, we would not observe synchrotron emission where the magnetic field presents significant values.

The coefficient in (113), $I \propto B^{7/2}$, is not perfectly known because it depends on the ratio of protons to electrons in cosmic rays, which has a value in the range 1-100, but following current estimates (Lisenfeld et al. 1996) for a typical VLA beam of 15 arcsec^2, 2.6 μJy would correspond to 1μG. However, the confusion limit, or minimum detectable flux at, say, 1.5 GHz is about 20 μJy, noticeably larger than the expected 2.6μJy.

Some works take the equation (112) with a hypothesis about N_0. If relativistic electrons are born in type-II Supernova explosions, which in turn are produced in regions of star formation, i.e. in sites with high gas density, and if relativistic electrons are not able to travel far from the birth region, then $N_0 \propto \rho$, could be an interesting, simple and acceptable assumption. But in this case, the radial decrease of I would be much faster; much faster even than the exponential (with typical radial scale length about 3 kpc). The reduction of ρ because of the external flaring would give a still faster truncation of the synchrotron continuum. If we assume that type-I Supernovae also contribute to producing relativistic electrons, the truncation of I will be even faster, as a result of the stellar truncation typical in all disks. In the Milky Way it takes place at about 12 kpc (Porcel, Battaner and Jiménez-Vicente, 1997).

Moreover, there is another argument to show that the absence of synchrotron radiation does not imply the absence of magnetic fields. It is observed that the synchrotron spectrum suddenly steepens for large radii. This feature takes place, for instance, in NGC 891 (Hummel et al, 1991; Dahlem, Dettmar and Hummel 1994) for $\geq$6 kpc. If the slope of the $[\log I, \log \nu]$ curve, usually called γ, is high, the number of very high energy electrons is relatively low. It is known (Lisenfeld et al. 1996) that these very high energy electrons have less penetration capacity, i.e. they cannot travel far from their sources. The simplest form of interpreting the increase of γ at those radii when the synchrotron becomes undetectable is a truncation of the relativistic electron sources. It is then probable that, the absence of cosmic electrons, rather than the absence of magnetic fields, is responsible for the low synchrotron intensity in the outermost disk.

4.4.2 The magnetic model

In this Section we try to determine what kind and magnitude of magnetic fields are necessary to explain the rotation of the outermost disk. These magnetic fields could introduce some instabilities in the disk, related to flaring, winds and escape, which are also examined.

The one-dimension model This exploratory model was developed by Battaner et al. (1992) following the magnetic hypothesis previously proposed by Nelson (1988). The two models are not equivalent. For instance, Nelson's magnetic field must have a non-vanishing pitch angle (i.e. the angle between the direction of the field -assumed to be contained in the galactic plane- and the azimuthal direction). Battaner et al. (1992) consider pure azimuthal (toroidal) fields or, rather, the azimuthal component of the field producing the required force.

In the radial component of the equation of motion, it is necessary to include magnetic forces, which are of the form $\frac{1}{4\pi}\vec{B}\cdot\nabla\vec{B}-\frac{1}{8\pi}\nabla B^2$ (e.g. Battaner (1996)). The first term in cylindrical coordinates (R, φ, z) will be

$$\frac{1}{4\pi}\vec{B}\cdot\nabla\vec{B} = \frac{1}{4\pi}(B_R, B_\varphi, B_z)\begin{bmatrix} \frac{\partial B_R}{\partial R} & \frac{\partial B_\varphi}{\partial R} & \frac{\partial B_z}{\partial R} \\ \frac{1}{R}\frac{\partial B_R}{\partial \varphi} - \frac{B_\varphi}{R} & \frac{1}{R}\frac{\partial B_\varphi}{\partial \varphi} + \frac{B_R}{R} & \frac{1}{R}\frac{\partial B_z}{\partial \varphi} \\ \frac{\partial B_R}{\partial z} & \frac{\partial B_\varphi}{\partial z} & \frac{\partial B_z}{\partial z} \end{bmatrix} \tag{115}$$

if the field is azimuthal, ($B_R = B_z = 0$), under azimuthal symmetry, ($\partial/\partial\varphi = 0$), the radial component of this force is simply $-B_\varphi^2/R$.

The magnetic pressure gradient force $-\frac{1}{8\pi}\nabla B^2$ would have a radial component simply given by $-(1/8\pi)\partial B_\varphi^2/\partial R$, and therefore the radial component of the magnetic force would be

$$F_{m,R} = -\frac{1}{4\pi}\frac{B_\varphi^2}{R} - \frac{1}{8\pi}\frac{\partial}{\partial R}B_\varphi^2 = -\frac{1}{8\pi}\frac{1}{R^2}\frac{\partial(R^2 B_\varphi^2)}{\partial R} \tag{116}$$

By including this force in the radial component of the equation of motion, we obtain

$$\rho\left[-\frac{\partial\mathcal{F}}{\partial R} + \frac{\theta^2}{R}\right] - \frac{\partial p}{\partial R} - \frac{1}{8\pi R^2}\frac{\partial(R^2 B_\varphi^2)}{\partial R} = 0 \tag{117}$$

in which steady-state conditions and vanishing viscosity are assumed. $\mathcal{F}$ is the gravitational potential. Here, the pressure gradient force in the radial direction is usually ignored.

From this equation, with current estimates for the different terms, but excluding any dark matter halo, it is easy to integrate numerically and obtain $B_\varphi(R)$. This was done by Battaner et al. (1992).

For didactic purposes only, it could be interesting to consider an ideal analytical calculation, assuming, for such large radii, that gravitation itself could be considered negligible. In such a case

$$B_\varphi^2 = \frac{B_0^2 R_0^2}{R^2} + 8\pi\theta^2\int_{r_0}^{R}\rho R dR \tag{118}$$

where $B_0 = B_\varphi(R = R_0)$ and R_0 is the radius where the integration begins. As ρ decreases exponentially (or faster, due to flaring, truncation, etc.) the last integral converges. For large enough R the first term would become dominant and we obtain that B_φ should become asymptotically

$$B_\varphi(R\to\infty) \to B_\varphi^*(R) \propto R^{-1} \tag{119}$$

This profile, $B_\varphi^*(R)$, does not produce any force, neither inwards nor outwards, and will be called critical. The real profile should have a slope lower than the critical one, to produce the fast rotation, as the magnetic pressure gradient force is probably an outward force. On the other hand, the magnetic tension B_φ^2/R is always inward and does not depend on the gradient, $\frac{\partial B_\varphi}{\partial R}$.

An intuitive reasoning underlying these equations, about how a magnetic tension produces an inward force, is that the term $1/4\pi\vec{B}\cdot\nabla\vec{B}$ pushes the gas along the field lines. In a ring where the magnetic field lines are circular and contained in the ring, a radial inward force will be produced. This force will also be present in the disk composed of many rings and if it is higher than that produced by the magnetic pressure, a net inward force would be added to gravity, which can only be compensated with an enhanced centrifugal force. Therefore the disk must rotate more rapidly.

In the exploratory model by Battaner et al. (1992), the calculated strengths are close to the critical (or asymptotic) profile $B^*_\varphi(R) \propto R^{-1}$ for very large radii. In this basic model a strength of about 6 μG at $R = 25kpc$ was obtained, which is indeed very high. In more recent models, which will commented later, much lower values of B_φ are obtained, even less than 1μG . The authors considered that the predicted synchrotron radiation was not in conflict with observations reported by Beck (1982) and that stability problems in the disk could arise if the strength reached such high values.

To explore this problem, Cuddeford and Binney (1993) developed a single model, with which they demonstrated that a disk with such a large magnetic field would produce excessive flaring. They assumed in this model that the magnetic pressure was α times the density, and this constant, α, was considered independent of z, but it was allowed to have a dependence on R. There exist several observations that prove that $B^2/8\pi$ decreases with z much more slowly than does the density, mainly in galaxies with a radio-halo (Ruzmaikin, Shukurov & Sokoloff, 1988; Hummel et al. 1989; Hummel, Beck & Dahlem, 1991; Breitschwerdt, McKenzie & Völk, 1991; Wielebinski, 1993; Han & Qiao, 1994 ...) and so the large flaring calculated by Cuddeford and Binney was clearly overestimated. Nevertheless, this paper was very illustrative in showing that the vertical component of the motion equation must unavoidably be integrated with the radial one, to assess the problem properly. If the magnetic field strength capable of driving the rotation curve is too high the disk may become thicker and flare. Moreover, Vallée (1997) considers that the strength required would unacceptably expand the HI disk.

But even when adopting $B^2/8\pi = \alpha\rho$ with $\alpha(r)$ being independent of z (i.e. the condition assumed by Cuddeford and Binney, 1993), the flaring of the disk would not be so large as estimated by these authors. If the disk were too thick the gas far from the plane would escape if it were slightly perturbed by very small vertical winds. As such winds are probably present in spiral galaxies, gravitation at such a high z would be too weak to retain gas moving outward.

An expanded disk cannot be retained. Clouds very far from the plane would be blown away by instabilities producing vertical winds and strong disk-corona interaction.

To demonstrate this and solve the problem raised by Cuddeford and Binney (1993) and Vallée (1997), Battaner and Florido (1995) developed a second model in which they adopted the most disadvantageous magnetic vertical profile, according to the $B^2 \propto \rho$ condition, but considered vertical winds and escape of gas, in the z-direction, to a galactic corona. Let us include a summary of this model.

The two-dimension model. Flaring and escape In this model by Battaner and Florido (1995) the vertical component of the equation of motion is written as

$$\rho v_z \frac{\partial v_z}{\partial z} + \left(\frac{v_A^2}{2} + \beta\right) \frac{\partial \rho}{\partial z} + \rho G M R^{-3} z = 0 \tag{120}$$

where $v_z(R, z)$ is the vertical velocity, $v_A(R)$ is the Alfven velocity considered to be a constant in the vertical direction (because of the assumption $B^2 \propto \rho$, as $v_A = B/\sqrt{4\pi\rho}$); also $\beta = p(R, z)/\rho(R, z)$ is assumed to be constant (isothermal condition, or rather, constant cloud-to-cloud velocity dispersion) and the vertical component of the gravitational force is simplified to that due to a central point mass; this assumption is in part justified because it is assumed that no dark matter halo is present and because we are considering just the outermost part of the disk. Many of these simplifying conditions are not necessary and, indeed, are only justified in a model with exploratory aims.

Therefore the motion in the vertical direction was considered to be the result of four terms: the inertial term, the magnetic, the gravitational and the pressure gradient forces. Horizontal velocities were considered, but the term $v_R \partial v_z/\partial z$ was neglected.

The equation of continuity restricts the possibilities of the vertical flux. It was assumed that

$$\rho(R, z) v_z(R, z) = \rho(R, 0) v_z(R, 0) \tag{121}$$

To calculate $v_z(R, 0)$ at $z = 0$ in the plane (or slightly above the plane, to avoid the problem arising from the symmetry in the galactic plane; $v_z(R, \Delta z) = -v_z(R, -\Delta z)$, for very small Δz, this would imply $v_z(R, 0) = 0$) some assumptions must be made; i.e. adopting a physical mechanism responsible for the vertical flux, which could be produced by supernova explosions. In this case it would be preferable to adopt a hypothesis of the type $v_z(R, 0) \propto \rho$, if for large time scales supernovae are born where the gas is denser. Battaner and Florido, instead, considered that Parker instabilities were the main origin of the vertical flux and assumed for the flux at $z = 0$:

$$\rho(R, 0) v_z(R, 0) = k \frac{B^2(R, 0)}{8\pi} \tag{122}$$

the value of k being considered a free parameter. This condition is equivalent to

$$v(R, 0) = k \frac{v_A^2(R)}{2} \tag{123}$$

Instead of ρ, it is preferable to use

$$y(R, z) = \frac{\rho(R, z)}{\rho(R, 0)} \tag{124}$$

because the calculation of $\rho(R, 0)$ and the profile $y(R, z)$ were made using different equations. Using Alfven's velocity the radial component of the equation of motion is now written as

$$\frac{1}{2}\frac{\partial}{\partial R}v_A^2(R) + \frac{1}{2}v_A^2(R)\left[\frac{1}{\rho(R,0)}\frac{\partial\rho(R,0)}{\partial R} + \frac{2}{R}\right] - \frac{v_H^2(R)}{R} = 0 \qquad (125)$$

The variable $v_H(R)$ is defined by

$$\frac{v_H^2(R)}{R} = \frac{\theta^2(R)}{R} - \frac{\partial\mathcal{F}(R)}{\partial R} \qquad (126)$$

which is what in more conventional theories is called the "halo velocity" as a way of introducing the halo potential. This quantity $v_H(R)$ was used because it can be found directly in the literature, but no dark matter halo was introduced. No inertial terms, neither $v_z\partial v_R/\partial z$ nor $v_R\partial v_R/\partial R$, were considered to be important and the gradient pressure force was again considered negligible.

The surface density was adopted from the literature, and therefore $\int_{-\infty}^{\infty}\rho(R,z)dz$ was kept constant. To compensate for the escape, it was assumed that the horizontal flux from the central region was so easily established that it was able to supply the necessary escaped mass at all radii. The function $v_R(r)$ was however found to be negligible. When the vertical velocities reached a value higher than the typical velocity dispersion, say 8 km/s, gas clouds were considered to have escaped from the disk, but not necessarily from the galaxy.

Though not affecting the numerical computations, it is interesting to obtain two functions of interest in the interpretation. One of these is the "Flaring Function", $Z(R)$ defined by

$$\int_0^{Z(R)}\rho(R,z)dz = \frac{1}{2}\int_0^{\infty}\rho(R,z)dz \qquad (127)$$

and the other is the total mass loss rate

$$\dot{M} = 2\int_0^{\infty}\rho(R,0)v_z(R,0)2\pi RdR = \frac{1}{2}\int_{R_0}^{\infty}kB^2(R)RdR \qquad (128)$$

where R_0 is the adopted inner boundary radius. $Z(R)$ is important because if the disk is highly magnetized, $Z(R)$ can become unacceptably large. If $\dot{M}$ is calculated to be too large, the whole galaxy could evaporate.

A numerical integration of these equations was carried out by Battaner and Florido (1995) taking M31 as a representative galaxy, with a convergent procedure that we do not reproduce here, but just show the results (see Fig. 20).

The free parameter k should have a value of between 10^{-9} and $10^{-8}cm^{-1}s$, with $k = 3\times10^{-9}$ being the value giving the most reliable results. The mass loss rate, $\dot{M}$ was found to be 0.054, 0.16 and 0.55 $M_\odot yr^{-1}$ for $k = 10^{-9}$, 3×10^{-9} and $10^{-8}cm^{-1}s$; this is still rather low, lower in any case than the typical value given by fountain models (15 $M_\odot yr^{-1}$; Kahn, 1994). Part of this gas that escaped from the disk would eventually fall back into the disk. Even if this were not the case and in the worst situation in which all the gas escaping from the disk escaped from the galaxy, the total mass loss during the whole history of the galaxy (assuming the flux to be constant in time) would be of the order of $0.16 M_{\odot} yr^{-1} \times 10^{10} yr \sim 1.6 \times 10^{9} M_{\odot}$, an acceptable value.

Even with the simplifying conditions assumed, a coherent general scenario is obtained:

a) The Alfven speed increases outwards is always lower than the rotational velocity but has a common order of magnitude. B and ρ decrease but B^2/ρ increases.

b) The effect of flaring and escape reduces the magnetic field required to drive the rotation of the outer disk. In the previous simple model, $6\mu G$ at 30 kpc was obtained. Now it is a full order of magnitude less. This is a very exciting figure, as it confirms that moderate magnetic field strengths can have a decisive influence on the rotation curve.

c) The flaring seems to be high but this is in reasonable agreement with observations. For low radii, the adoption of the central point mass potential is not appropriate. For instance, Z(R) is too high for radii less than about 17 kpc. But in our Galaxy, where precise data exist for very large radii, a value of $Z \sim 6$ kpc at R= 20 kpc from the plots provided by Diplas and Savage (1991) is reproduced by the model fairly well.

d) The densities are reasonable. At 30 kpc, values of the order of $1.6 \times 10^{-28} gcm^{-3}$ (10^{-4}atoms cm^{-3}) were obtained, and at 25 km, $5 \times 10^{-28} gcm^{-3}$. In any case these values are compatible with the observed surface density, as this function was adopted from the observations.

e) Velocities (vertical at the base of the galactic plane and radial) are small, of the order of a few km/s, which are nearly undetectable and therefore do not introduce problems of disagreement with any measurement. Consider that 2 km/s at $z \sim 0$ may produce 10 km/s at $z \sim 8kpc$ and $R \sim 20kpc$, due to continuity. (The flux would be conserved, so the decrease in density accelerates the vertical speed). Velocities of this order of magnitude are observed even in a quiet disk (see for instance Jiménez-Vicente et al. 1999).

Some indirect arguments against and favouring the magnetic hypothesis Apart from the objection by Cuddeford and Binney (1993) to the one-dimension model above mentioned, which was solved in the two-dimension model, Persic and Salucci (1993) considered the magnetic hypothesis as "neither necessary nor sufficient". Setting aside the question of how a theory can be "not necessary", they argued that galaxy pairs need dark matter, but we have seen that galaxy pairs admit other interpretations, in particular that of common halos.

Sánchez-Salcedo (1996) has considered the possibility that a relation found by Bosma (1978, 1981, 1993) between HI and dark matter density could be explained under the magnetic hypothesis. On qualitative grounds this would be reasonable, because the higher the gas density, the higher the magnetic strength that could be amplified.

There is a possible connection between the truncation of stellar disks and the magnetic hypothesis for the rotation curves. Once stars are born, the centripetal magnetic force, previously acting on the progenitor gas cloud, is suddenly interrupted and stars move to larger orbit radii or escape. This escape would be responsible for the truncation of stellar disks, which is a common feature in spirals.

Vallée (1994, 1997) also addresses this point. He considered that newly formed stars would acquire ballistic velocities of the order of the rotation velocity of the parent gaseous cloud. Stars with this velocity have not been observed. If stars were rotating in Keplerian orbits -Vallée argues- they should decelerate. The answer to this problem raised by Vallée (1997) lies in the fact that many new-born stars could escape. Others would simply migrate to more energetic orbits. A numerical model of high velocity new-born stars, escape in the radial direction and truncation of the stellar disk is currently being constructed.

Pfenniger, Combes and Martinet (1994) and Jopikii and Levy (1993) argued that, following the Virial theorem, magnetic fields should have an expansive effect, in contrast with the magnetic centripetal force deduced by Battaner et al. (1992). This is a peculiar argument, as the Virial theorem is deduced from the equation of motion, which was the equation integrated by Battaner et al. (1992). This could constitute a paradox in the one-dimension model. The solution should be found in the two-dimension model (Battaner and Florido, 1995), where it was shown that there is an escape of gas in the vertical direction. In other words, magnetic fields have a contracting effect in the radial direction, but an expansive one in the direction perpendicular to the disk; hence, the net effect of magnetic fields could be expansive. We have also suggested that a large fraction of new-born stars could escape in the radial direction, which is also an expansive dynamical effect. It should be borne in mind that the two-dimension model (1995) was published after the study of Pfenniger, Combes and Martinet (1994).

The inclusion of magnetic fields by Nelson (1988), Battaner et al. (1992) and others have -in the opinion of Pfenniger, Combes and Martinet- "the implicit hope that by complicating the physics new alternatives can emerge". However, magnetic fields were introduced in Physics several centuries ago, while the inward force due to the magnetic tension in a magnetized disk is a conclusion of really elementary physics.

4.4.3 Mechanisms producing magnetic fields in the outermost disk

Magnetic fields may explain rotation curves if a) there is a sub-critical strength gradient and b) they have a sufficient order of magnitude. The next step is to deduce the existence of these fields theoretically and to identify the mechanisms that produce them.

A first simple model with this objective was presented by Battaner, Lesch and Florido (1999), in which a mechanism is responsible for a critical slope, $B_\varphi \sim B_\varphi^* \propto R^{-1}$. A highly convective disk in the vertical direction maintains a highly turbulent magnetic diffusivity, establishing a connection and equilibrium between extragalactic and galactic fields. The origin of galactic fields is extragalactic and they are amplified and ordered by differential rotation. The problem of the origin of magnetic fields is then shifted to the intergalactic medium, a topic that will be addressed in the next section.

With this model, we depart from the classical approach, basically consisting of the $\alpha\Omega$ dynamo or similar models. We can allow ourselves this liberty because the classical dynamo theory (summarized, for instance, in the review by Wielebinski and Krause, 1993) has been subject to severe criticism and does not offer a clear scenario. The standard dynamo approach does not take into account the back reaction of the turbulence on the amplified magnetic field, which is very strong at small scales (Kulsrud, 1986; Kulsrud and Anderson, 1992). Another important shortcoming of the standard dynamo theory lies in the following fact: The $\alpha\Omega$ dynamo exponentially amplifies a preexisting seed field up to the present, with strengths of the order of 1-10 μG. The field is amplified e-times in each rotation. Suppose that the galaxy has rotated about 20 times since its birth. Then, the field has been amplified by a factor of about $e^{20} \simeq 5 \times 10^8$. Therefore, the initial strength would have been about 10^{-15}G. This is in contradiction with the μG fields measured in 3C295 (with z = 0.395) (Kronberg, Perry and Zukowski, 1992). Moreover Perley and Taylor (1991) detected such large fields at z=0.461. Absorption Line Systems of quasar spectra, usually interpreted as pregalactic structures, also have μG fields (Kronberg and Perry, 1982; Watson and Perry, 1991). Observations of Lyman-α clouds at $z \sim 2$ also show $\sim 3\mu G$-fields (Wolfe, Lanzetta and Oren, 1992), similar to other highly redshifted disks (Wolfe, 1988; Kronberg et al., 1992). If new-born galaxies were so highly magnetized, the $\alpha\Omega$ dynamo would have amplified these initial fields to a present value of about 500 G, in astonishing disagreement with observations. Even if, before reaching this value, some saturation mechanism had appeared, the classical dynamo is incompatible with pregalactic μG-strengths. Therefore, the topic is now free for speculation and the search for alternative scenarios.

This argument not only invalidates the classical dynamo theory but also many hypotheses about the origin of primordial magnetic fields that were conceived as providing $\sim 10^{-15}$G at the epoch of galaxy formation. Galaxies were probably formed out of an already strongly magnetized medium, with an equivalent-to-present $\sim 1\mu G$ field, the same order of magnitude as the present intergalactic medium field.

Turbulent difussion and the galactic magnetic field In this Section let us summarize the work by Battaner, Lesch and Florido (1999). The similarities of the strengths in the interstellar, the extragalactic and the pregalactic media suggest a fast and efficient connection between them. In this work, it is proposed that this connection is the result of a highly turbulent magnetic diffusion in the vertical direction.

It is an observational fact that convective phenomena are very active in disks. Galaxies sometimes exhibit “boiling” disks, with NGC 253 being a good example (Sofue, Wakamatsu and Malin, 1994), where dark filaments, lanes, arcs and other micro-structures are features revealing a very complex convective region. This fact is in part explained by “fountain” models (Shapiro and Field, 1976; Breitschwert et al., 1991; Kahn, 1994; Breitschwerdt and Komossa, 1999). Of course, these turbulent motions constitute a transporting of magnetic fields, as a result of the condition of frozen-in lines. Because of this transporting, extra and intergalactic fields merge.

Suppose first that no dynamo is acting on the galactic gas. The equation of induction will tell us (e.g. Ruzmaikin, Shukurov and Sokoloff, 1988; Battaner, 1996)

$$\frac{\partial B_R}{\partial t} = \beta \frac{\partial}{\partial z}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right) \tag{129}$$

$$\frac{\partial B_\varphi}{\partial t} = B_R \frac{\partial \theta}{\partial R} + B_z \frac{\partial \theta}{\partial z} - \theta \frac{B_R}{R} + \beta \left(\frac{\partial^2 B_\varphi}{\partial z^2} + \frac{\partial}{\partial R}\left(\frac{\partial B_\varphi}{\partial R} + \frac{B_\varphi}{R}\right)\right) \tag{130}$$

$$\frac{\partial B_z}{\partial t} = -\beta \left(\frac{\partial}{\partial R}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right) + \frac{1}{R}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right)\right) \tag{131}$$

where B_R, B_φ and B_z are the magnetic field strength components and β is the coefficient of turbulent magnetic diffusion. Cylindrical symmetry has been assumed. The usual expression to calculate β is

$$\beta = \frac{1}{3} vl \tag{132}$$

where l is a typical length of the larger convective cells, say $l \approx 1kpc$, and v is a typical convection velocity corresponding to the larger scale turbulence, say 20 km s^{-1}. Hence β is of the order of $2 \times 10^{27} cm^2 s^{-1} \approx 6 kpc^2 Gyr^{-1}$. In comparison, β is taken as being of the order of $10^{26} cm^2 s^{-1}$ in the inner disk), of $5 \times 10^{27} cm^2 s^{-1}$ in the galactic corona and of $8 \times 10^{29} cm^2 s^{-1}$ in the intergalactic medium in a cluster (Ruzmaikin, Sokoloff and Shukurov, 1989; Sokoloff and Shukurov, 1990; Ruzmaikin, Shukurov and Sokoloff, 1988).

The characteristic diffusion time is calculated with l^2/β, therefore having a typical value of 0.2 Gyr, very little compared with the lifetime of the galaxy. Extragalactic magnetic fields would have spatial variations at scales much larger than a galaxy. The field strength can be assumed to be constant outside the galaxy, as a boundary condition. This external steady state magnetic field could have produced an initial penetration of magnetic fields which would have been subsequently ordered by differential rotation, resulting in a predominantly toroidal field. Or rather, the disk was born out of already magnetized material, then was magnetized at birth and maintains a permanent interchange with the magnetized environment, because of the high magnetic diffusivity.

However, the magnetic field is assumed to be homogeneous outside and toroidal inside. A configuration that continuously transforms a constant into a toroidal field was proposed by Battaner and Jiménez-Vicente (1998), but here we need to adopt convenient boundary conditions taken at a large enough height.

All three components -B_x, B_y, B_z- are constant in the extragalactic medium. But not all penetrate and diffuse inwards equally. For instance, there is no difficulty for B_z to penetrate, because it is not perturbed by rotation. And if the transport is so effective we could even assume that B_z is a constant in the whole outer disk considered, equal to the extragalactic value of B_z. We then assume as a reasonable mathematical assumption that B_z=cte everywhere in the integration region.

It is more difficult for the other components to penetrate (or exit). For instance, B_R penetrates into the disk at a given time and point (R, φ); the rotation would transport the frozen-in magnetic field lines into the azimuthally opposite position (R, $\varphi + \pi$) in half a rotation period. The direction of the penetrated field vector there would be opposite to the vector transported from the opposite azimuth. The two vectors would meet with the same modulus and opposite direction and would destroy one another through the reconnection of field lines. It is therefore tempting, in a first simplified model, to assume that $B_R = 0$, at the boundaries. We may even adopt $B_R = 0$ everywhere inside the disk.

With respect to B_φ, we have a similar situation. B_φ when penetrating at (R, φ) would be frozen-in transported to (R, $\varphi + \pi$) in half a rotation and then interact with the field penetrated there. Reconnection would then act and we could reasonably adopt $B_\varphi = 0$ at the boundaries. But B_φ is easily amplified by rotation and can be generated from B_z, which is non-vanishing; therefore we cannot assume $B_\varphi = 0$ everywhere; rather it is $B_\varphi(R, z)$ that we want to calculate. We also assume steady-state conditions, $\partial/\partial t = 0$. The equations then become greatly simplified which also permits a simplified interpretation of what is essential in the process, much more understandable than a lengthy numerical calculation. In the above equations, we set $\partial/\partial t = 0$, B_z=constant, $B_R = 0$ and obtain

$$0 \equiv 0 \tag{133}$$

$$0 = B_z \frac{\partial \theta}{\partial z} + \beta \left(\frac{\partial^2 B_\varphi}{\partial z^2} + \frac{\partial}{\partial R} \left(\frac{\partial B_\varphi}{\partial R} + \frac{B_\varphi}{R} \right) \right) \tag{134}$$

$$0 = 0 \tag{135}$$

The first and third are tautologic, telling us that we could have deduced much of what was assumed (as Battaner, Lesch and Florido did), but that is unimportant. We now see that our assumptions do not lead to incoherent results.

The second equation would provide us with B_φ if $\theta(z)$ were known. Let us further assume $\partial\theta/\partial z = 0$, which is not unrealistic given the relatively low thickness of the disk. In order to find a fast solution (which is not necessary, but just didactic) let us assume that $\partial^2 B_\varphi/\partial z^2$ is negligible (it cannot be zero, as B_φ must be zero at the boundary). In fact, some galaxies have a radio halo (e.g. NGC 253, Beck et al. 1994; NGC 891, Dahlem, Lisenfeld and Golla, 1995 in other spirals). The decrease of magnetic field strength with z is observed to be very slow, even in galaxies with no radio halo (Ruzmaikin et al., 1988; Wielebinski, 1993) and also in the Milky Way (Han and Qiao, 1994). Then for small $\mid z \mid$ we simply obtain

$$\frac{\partial}{\partial R}\left(\frac{\partial B_\varphi}{\partial R} + \frac{B_\varphi}{R}\right) = 0 \tag{136}$$

therefore

$$B_\varphi \propto \frac{1}{R} \tag{137}$$

which is precisely the critical profile. Once we see how the critical profile is supported with this mechanism, it is expected that other more realistic calculations would be able to provide sub-critical profiles, capable therefore of producing inward magnetic forces.

The symmetries of the magnetic field predicted in this simple model agree with those obtained with Faraday rotation by Han et al. (1997) in our own galaxy. This model does not need a dynamo but provides a large-scale structure with much in common with the so called AO mode. We also predict an antisymmetry of the azimuthal field in both hemispheres for $\mid l \mid < 90^o$. This AO dynamo mode has also been observed in other galaxies, but in view of the symmetry similarities with our predictions, these galaxies could be interpreted in terms of the mechanisms sought by Battaner, Lesch and Florido (1999).

A dynamo-like mechanism Though we favour the previous model based on the action of turbulent magnetic diffusion, let us show that there is also a special kind of dynamo or amplification mechanism that, in contrast with the standard $\alpha\Omega$, quickly reaches steady state conditions, and that also produces the critical profile of the magnetic field strength.

Let us continue considering azimuthal symmetry. In addition to β terms, let us consider α terms, i.e. the effect based on a mean value of the quantity $< \vec{v} \cdot \nabla \times \vec{v} >$, non vanishing in turbulence velocity fields, because the Coriolis force introduces order into the chaotic turbulence. With this symmetry the induction equation becomes

$$\frac{\partial B_R}{\partial t} = -\alpha\frac{\partial B_\varphi}{\partial z} + \beta\frac{\partial}{\partial z}\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial R}\right) \tag{138}$$

$$\frac{\partial B_\varphi}{\partial t} = B_r\frac{\partial \theta}{\partial R}+B_z\frac{\partial \theta}{\partial z}-\theta\frac{B_R}{R}+\alpha\left(\frac{\partial B_r}{\partial z}-\frac{\partial B_z}{\partial R}\right)+\beta\left(\frac{\partial^2 B_\varphi}{\partial z^2}+\frac{\partial}{\partial R}\left(\frac{\partial B_\varphi}{\partial R}+\frac{B_R}{R}\right)\right) \tag{139}$$

$$\frac{\partial B_z}{\partial t} = \alpha\left(\frac{\partial B_\varphi}{\partial R}+\frac{B_\varphi}{R}\right)-\beta\left(\left(\frac{\partial}{\partial R}\left(\frac{\partial B_R}{\partial z}-\frac{\partial B_z}{\partial R}\right)\right)+\frac{1}{R}\left(\frac{\partial B_R}{\partial z}-\frac{\partial B_R}{\partial R}\right)\right) \tag{140}$$

If, as in the previous Section, we set $B_R = 0$, B_z constant, and assume steady state conditions, this equation system reduces to

$$0 = \alpha\frac{\partial B_\varphi}{\partial z} \tag{141}$$

$$0 = B_z\frac{\partial \theta}{\partial z}+\beta\left(\frac{\partial^2 B_\varphi}{\partial z^2}+\frac{\partial}{\partial R}\left(\frac{\partial B_\varphi}{\partial R}+\frac{B_R}{R}\right)\right) \tag{142}$$

$$0 = \alpha\left(\frac{\partial B_\varphi}{\partial R}+\frac{B_\varphi}{R}\right) \tag{143}$$

The first equation confirms our previous assumption. The third equation is very illustrative, as, even if $\beta = 0$, in the absence of significant turbulent magnetic diffusion, we again obtain the critical profile $B_\varphi = B_\varphi^* \propto \frac{1}{R}$. As the first one informs us that $\partial B_\varphi/\partial z$, then the second just tells us that this solution is compatible with either $B_z = 0$, or no vertical differential rotation $\frac{\partial \theta}{\partial z} = 0$. It is important to note that we have obtained the critical profile both without β and including β. In contrast with other dynamos, this mechanism can reach a steady state.

4.4.4 The origin of cosmic magnetic fields

We now consider the origin and evolution of cosmological magnetic fields. Our aims are: a) To classify the long list of theories which have been proposed; b) To provide astrophysical arguments constraining these theories, reducing their number, if possible; c) To propose or adopt a coherent overall history of cosmological magnetic fields; d) To determine whether this general scenario provides a reasonable basis for the magnetic hypothesis of rotation curves.

An important argument severely reducing the long list of candidate theories has been given by Lesch and Birk (1998), in which they prove that small scale magnetic fields cannot survive the highly resistive pre-recombination era, characterized by frequent electron-photon interactions. Therefore, cosmic magnetic fields, were either generated after Recombination or had coherence cells larger than the horizon before Recombination. In this latter case, diffusion of magnetic fields cannot proceed over distances larger than the horizon, and these cells could become a subhorizon in the post-Recombination epoch, which is more favourable for the existence of magnetic fields.

Let us then place the limit between the large and small scales at about 10 Mpc, because this is the minimum length that was superhorizon before (about)

Recombination, or more precisely before "Equality" (the transition epoch between the radiation and matter domination). Therefore, magnetic field coherence cells longer than (today) $\sim$ 10 Mpc were not set in motion by diffusion in the resistive era before (about) Equality. Battaner, Florido and Jiménez-Vicente (1997) and Florido and Battaner (1997) observed a clear transition in the evolution of magnetic fields for scales

$$\lambda > \frac{1}{mn_0}\sqrt{\frac{3\sigma T_0^4}{8\pi c G}} \tag{144}$$

where mn_0 is the present cold matter density (baryonic or not; dark or not), σ is the Stephan-Boltzmann constant and T_0 the CMB temperature. This length is equivalent to just a few Mpc.

This transition is very important for our purposes as large scale fields are not influenced by diffusion or any other micro-physical effect during the radiation dominated era.

Reviews of cosmological magnetic fields have been written by Rees (1987), Coles (1992), Enquist (1978), Olesen (1997), Vallée (1997) and others. Closely related to this topic are the works by Zweibel and Heiles (1997) and by Lesch and Chiba (1997).

In the absence of loss and production amplification mechanisms, frozen-in magnetic field lines will evolve due to the flux of Hubble alone as

$$\vec{B}_0 = \vec{B}a^2 \tag{145}$$

where a is the cosmological scale factor, taking its present value as unity, $\vec{B}$ the magnetic field strength when the Universe was a times smaller than today and $\vec{B}_0$ the present strength. This equation is in general not true, because the frozen-in condition is not guaranteed at all epochs, and because production, loss and amplification processes other than those due to the Hubble flow could really have taken place. However, we will adopt this equation, as a re-definition of $\vec{B}_0$. The equivalent-to-present magnetic field strength, $\vec{B}_o(t)$, is the strength that would be observed today corresponding to the real $\vec{B}_0(t)$ when the Universe was $a(t)$ times smaller, as a result of frozen-in lines in the Hubble flow, in the absence of an amplifying or destroying mechanism other than the expansion itself, even if it does not coincide with the present one at all. As the effect of expansion is always important, this definition of the equivalent-to-present magnetic field strength permits a useful comparison of strengths during different epochs.

On the other hand, this expression can be more general (Battaner, Florido and Jiménez-Vicente, 1997) and holds under the condition of $\vec{B}$ constituting a small perturbation of the Robertson-Walker metrics. A pure U(1) gauge theory with the standard Lagrangian is conformally invariant (unlike a minimally coupled field), from which it follows that $\vec{B}$ always decreases in the expansion following this equation, even in the absence of charge carriers. The Inflation epoch may be an exception for the reasons given below.

Following Battaner and Lesch (2000), the different theories about the origin of cosmic magnetic fields can be classified into four main groups, characterized by the epoch of formation:

a) during Inflation
b) in cosmological phase transitions
c) during the Radiation Dominated era
d) after Recombination

Inflation magnetic fields The mean magnetic field of the Universe is zero, if we adopt the Cosmological Isotropy Principle, and therefore it would be random at the larger scales. At smaller scales, there can be coherence cells with different sizes, characterized by a mean field. Coherence cells are usually associated with objects or density inhomogeneities . It could therefore be expected that anisotropies in the CMB, which are associated with density inhomogeneities, might correspond to large magnetic coherence cells. Some anisotropies are larger than 2^o, i.e. they are larger than the horizon at Recombination. These larger inhomogeneities were generated before or during Inflation. In the same way, Inflation provides the most natural explanation of field inhomogeneities, as it permits a causal connection between two points with a separation that was until fairly recently (until Equality, approximately) smaller than the horizon.

Turner and Widrow (1988) were pioneers in analyzing this idea. A cloud with a present diameter λ, had in the past a size $a\lambda$. However, the horizon evolved independently of a during the first phase of Inflation, and was then proportional to $a^{3/2}$ during the so called reheating phase, to a^2 during the radiation dominated era, and to $a^{3/2}$ after Equality (when the radiative and the matter energy densities became equal). Therefore, a cloud could be subhorizon before or during Inflation, become superhorizon thereafter and again be subhorizon at present. This could explain how points in the cloud at distances further than the horizon before the present CMB anisotropy are causally connected. In the same way, magnetic coherence cells, causally connected at Inflation, could have become superhorizon early and reentered the horizon recently.

Coherence could even be due to electromagnetic waves, as the oscillating electric and magnetic fields, when the wavelength became subhorizon, would have appeared as static fields. Only recently, after around Equality, would conductivity have destroyed large scale electric fields and controlled large scale magnetic fields.

The problem inherent in this theory is that $a(t)$ was exponential during Inflation, increasing by a factor of 10^{21}. This would imply a decrease in B by a factor of 10^{42} (perhaps much more), if Ba^2 were a constant, i.e. if the U(1) gauge theory were conformally invariant. This dilution of magnetic fields must be avoided by some mechanism. Turner and Widrow considered that the conformal invariance of electromagnetism is broken through gravitational coupling of the photon. In this case, the electron would have a mass of only about 10^{-33}eV, and therefore be undetectable. Turner and Widrow predicted $B_0 \sim 5 \times 10^{-10}$G at scales of about 1 Mpc, which is really interesting.

Other authors avoided the complete dilution of primordial fields with other mechanisms. Ratra (1992) considered the coupling of the scalar field responsible for inflation (the inflaton) and the Maxwell field, obtaining $B_0 \sim 10^{-9}$G at scales of 5 Mpc. Garretson, Field and Carroll (1992) invoked a pseudo-Goldstone-boson coupled to electromagnetism ($B_0 < 10^{-21}$G at $\sim$1 Mpc). Dolgov (1993) proposed the breaking of conformal invariance through the "phase anomaly". Dolgov and Silk (1993) considered a spontaneous break of the gauge symmetry of electromagnetism that produced electrical currents with non-vanishing curl.

Davis and Dimopoulos (1995) considered a magnetogenesis at the GUT phase transition, but their theory is included here because this transition could have taken place during Inflation (10^{-11}G at galactic scales).

Rather interestingly, when considering the Planck era, the Superstring theory leads to an inflationary pre-Big-Bang scenario which supports some of the theories explained before (Veneziano, 1991; Gasperini and Veneziano, 1993a,b; Gasperini, Giovannini and Veneziano 1995a, b; Lemoine and Lemoine, 1995; etc.) rendering derivations from what were assumptions. In this scenario, the electromagnetic field is deduced to be coupled not only to the metric but also to the dilaton background. COBE anisotropies are the result of electromagnetic vacuum fluctuations, involving scales of the order of comoving 100 Mpc, today. For values of some arbitrary parameters, these models provide large enough values of intergalactic fields, even in the absence of galactic dynamos. They are in fact able to explain a possible equipartition of energy between the CMB radiation and magnetic fields. This pre-Big-Bang scenario is really promising as an explanation of primordial magnetic fields and their connection with CMB.

Phase transition magnetic fields During the history of the Universe several cosmological phase transitions have taken place. The best studied are: QCD (250 MeV), electroweak (10^2 GeV) and GUT (10^{16}GeV). For a comparison, consider that the present epoch is characterized by 3×10^{-4} eV, the matter-radiation decoupling by 1 eV and the nucleosynthesis epoch by 1 MeV. Typical values of the horizon scale correspond to a present-day 0.2 pc for the QCD transition, 3×10^{14} cm for the electroweak transition and 1 m for GUT.

Hogan (1983) proposed first order phase transitions as a potential magnetogenetic mechanism. Phase transitions would not have taken place simultaneously throughout the Universe, but in causal bubbles. At the rim of these bubbles high gradients of temperature or any other parameter characterizing the phase transition (for instance, the Higgs vacuum expectation value) would be established. These high gradients could produce a thermoelectric mechanism akin to the Biermann battery. Magnetic reconnection would stitch the magnetic field lines of different bubbles and the magnetic field lines would execute a Brownian walk.

There exist a large variety of works and ideas. The electroweak transition has been considered as a source of magnetic fields by Vachaspati (1989), Enqvist and Olesen (1993), Davidson (1996), Grasso and Rubinstein (1995) and others. The QCD phase transition as a magnetogenesis mechanism has been studied by Quashnock, Loeb and Spergel (1989), Cheng and Olinto (1994) and others. Magnetic fields generated at the GUT phase transition have been analyzed by Davies and Dimopoulos (1995), Brandenberger et al. (1992). Other interesting theories related to Cosmological phase transitions have been proposed by Vachaspati and Vilenkin (1991), Kibble and Vilenkin (1995), Baym, Boedeker and McLerran (1996) and others.

From the equations for magnetic fields produced at a given phase transition, and the spectra at different length scales given by Vachaspati (1989) it is deduced (Battaner and Lesch, 1999) that B, the equivalent-to-present magnetic strength, only depends on T_0^2 (where T_0 is the present CMB temperature) and that the spectrum $B_0(\lambda)$ only depends on T_0/λ (or on $T_0^{3/2}\lambda^{-1/2}$ with a correction proposed by Enquist and Olesen (1993)). In any case the present spectrum of magnetic strength for different scales $B_0(\lambda)$ is independent of the specific phase transition. There is one compensation: earlier phase transitions produced larger magnetic fields but they have had longer to be weakened by expansion. We will comment later that the values of B_0 can differ greatly from present magnetic field strengths.

Magnetic fields generated by turbulence during the radiation dominated era There are two classical papers (Matsuda, Sato and Takeda, 1971; Harrison, 1973) in which magnetic fields were considered to be generated by turbulence in the radiation dominated universe. There is also a close relation between vorticity and magnetic fields, $\vec{B} = -(mc/e)\vec{\omega}$ (where $\vec{\omega}$ is the vorticity) which was deduced by Batchelor (1950) and considered again by Kulsrud et al. (1997) as an extension of a previous study by Biermann. The deduction is based on the surprising similarity between the vorticity and the induction equations. With similar initial conditions, both magnitudes, vorticity and magnetic fields, should evolve similarly. Viscosity has a different behaviour, because there is a saturation of vorticity, but the above equation could be used in other astrophysical problems. In the epoch of radiation domination, however, we will present arguments against this, so that this equation probably does not hold.

Magnetic Fields generated after Recombination When dealing with cosmic magnetic fields in present day astrophysical problems, it is customary to fully assume hypotheses that are accepted in magnetohydrodynamics. It is in fact assumed that magnetic fields can be modified, amplified and even be subject to diffusion or reconnection, but that they cannot be created. However, the three above mentioned mechanisms are able to create magnetic fields out of nothing. There are also more classical mechanisms of net creation of magnetic fields, with the well known Biermann's battery providing a clear example (Biermann, 1950; Biermann and Schleuter, 1951). Another battery mechanism was proposed by Mishustin and Ruzmaikin (1973), in which the CMB radiation differentially interacts via Compton scattering with protons and electrons, thus establishing a weak electric field and weak electrical currents that in turn are able to originate weak magnetic fields.

In a protogalactic cloud, the conditions are similar to those needed for the classical Biermann's battery, mainly a combination of gravitational field with differential rotation. Lesch and Chiba (1995) showed that magnetic field strengths in the range $10^{-13}-10^{-16}$G can be produced at early stages in the protogalactic cloud. This seed may be exponentially amplified by non-axisymmetric instabilities during the disk formation epoch, so that magnetic fields of the order of 1μG can be reached in less than 1-2 Gyr, as observed in recently born galactic systems (Chiba and Lesch, 1994). Kulsrud et al. (1997) and Howard and Kulsrud (1996) also demonstrated that protogalactic magnetic fields can be created without any seed after Recombination.

Kronberg, Lesch and Hopp (1998) have proposed that superwinds of dwarf galaxies of the M82-type, which eject great quantities of matter and magnetic fields, have effectively seeded the intergalactic medium with magnetic fields, in a first generation of ($z \sim 10$) galaxies. The seeding would have been accomplished by $z \sim 6$. Under this hypothesis, pre-Recombination fields would not be required, at least to understand galactic fields.

Comments on the different theories Let us first discuss some problems inherent to theories based on phase transitions. Phase transition generated magnetic fields have small scales. For instance, the most recent one, the QCD transition, at ∼200 MeV, predicts correlation lengths of 10^{-11}cm, which grow to 10 cm at present. The horizon at the QCD phase transition was 10^{-6}cm, equivalent to 0.2 pc at present. Other phase transitions also predict small scales. The electroweak phase transition took place when the horizon was at only a few centimeters, corresponding to about 1 AU at present. For early phase transitions the expected scale is even worse.

These fields undoubtedly created in phase transitions probably have no connection with present magnetic fields, because there are two mechanisms that can destroy this kind of small scale fields.

First, the subsequent radiation-dominated universe was highly resistive, because of the frequent encounters between electrons and photons. This has been shown by Lesch and Birk (1998). The low conductivity implies magnetic diffusion. These authors gave a diffusion time equivalent to

$$\tau_{diff} = 10^{44} z^{-6} \lambda^2 \tag{146}$$

where τ_{diff} is measured in seconds and λ, the coherence size, in cm. This time very much depends on redshift, with the initial times being the most destructive, probably just after Annihilation, because of the increase in the photon number density. If we set $\tau_{diff} = \tau_{rec}$ (Recombination epoch) and $z = z_{ann}$ (Annihilation redshift) we will obtain the minimum scale able to survive from Annihilation to Recombination

$$\lambda = 5 \times 10^{-16} z_{ann}^3 \tag{147}$$

This λ will grow to its present comoving size

$$\lambda_0 = 5 \times 10^{-16} z_{ann}^4 \tag{148}$$

For $z_{ann} \sim 2 \times 10^9$, we conclude that the minimum scale able to survive was about 3 kpc, much higher than the scale predicted by the magnetogenesis mechanisms based on cosmological phase transitions.

It is understandable that magnetic fields, and density and radiation inhomogeneities are associated during the radiation dominated Universe. Therefore if matter or radiation overdensity regions, at a certain scale, are destroyed or damped, the same end should be expected for magnetic fields of this scale. It is known (Silk, 1968; Weinberg, 1968) that masses smaller than the Silk mass are damped in the Acoustic epoch, before Recombination. Jedamzik, Katalinic and Olinto (1996) also concluded that MHD modes are completely damped by photon diffusion up to the Silk mass and convert magnetic energy into heat. Damping would also be very important during the neutrino decoupling era. Therefore, small scale fields could have been eliminated before the radiation era.

Therefore, small scale fields, even if they were created in phase transitions, cannot survive the radiation dominated era. They have two enemies: magnetic diffusion and, probably, photon diffusion.

However, we must mention the important work by Brandenburg, Enqvist and Olesen (1996) in which they propose that inverse cascade effects in relativistic turbulence in the expanding medium produce large scales. Then, inverse cascade would save the small scale phase transition magnetic fields. But the existence of a turbulence during this epoch is controversial (Rees, 1987), or at least it would have had a very peculiar behaviour. In fact $\delta\rho/\rho$ cannot evolve in a random way. If $\delta\rho/\rho$ is small but positive, it will always increase and remain positive if the cloud mass is higher than the Jeans mass, because of gravitational collapse. The Jeans mass is very low, particularly just after Annihilation, of the order of 1 $M_\odot$, and therefore gravitational collapses, rather than true turbulence, dominated the evolution of initial inhomogeneities. Perturbations of the metric tensor are essential in this era. Even if the inhomogeneities do not grow very fast (as a^2) they cannot be neglected. On the other hand, turbulent motions, if they really existed, could not affect scales larger than the horizon, and therefore scales larger than 1 Mpc cannot be produced. In fact, Brandenburg, Enqvist and Olesen predict much lower amplification factors, given the initial very small scales to be enlarged.

These arguments seem to exclude phase transitions as mechanisms providing magnetic fields connected to present fields. Moreover, the model proposed by Harrison (1973) even if historically interesting, and emphasizing the effect of the horizon on the turbulence regime, did not include General Relativity effects, which are not ignorable at all.

Therefore, despite the possibilities of an extended analysis of inverse cascade effects, we favour the hypothesis that large scale fields (larger than the horizon in the radiation dominated era) were produced at Inflation, as deduced in the scenario of string cosmology (e.g. Gasperini and Veneziano 1993a,b). Small scale fields, such as those in galaxies, have two possible origins: a) The large scale inflation magnetic fields were amplified after Recombination as a result of contractions in the process of forming superclusters, clusters and galaxies after Recombination; b) They were generated without seeding by battery mechanisms in the process of galaxy formation.

Irrespective of the exact time and mechanism of magnetogenesis, the effect of preexisting magnetic fields on the birth and structural properties of galaxies has been considered in the literature. Piddington (1969) tried to explain the present morphology of different types of galaxies from the angle between the angular momentum and the magnetic field strength. Wasserman (1978) considered that the formation of galaxies was decided by preexisting magnetic fields and that these were even able to provide the galactic angular momentum. Kim, Olinto and Rosner (1996) extended this work to the non-linear regime.

It is difficult to decide between the two possibilities for the origin of small scale magnetic fields, and therefore to decide what is the origin of galactic magnetic fields. We prefer to argue in favour of the inflationary origin, for the following two reasons, one theoretical and the other observational:

a) We will see that magnetic fields of the order of $B_0 \sim 10^{-9} - 10^{-8}$G may be present in the ~ 100 Mpc long filaments characterizing the large scale structure of the Universe. These structures probably consisted of filamentary concentrations of photons, baryons and possibly other kinds of dark matter, but the energy density was smooth and continuous within a filament. After Recombination, baryons and dark-matter particles begun to form clumpy structures of a different order (superclusters, clusters, galaxies), and the contractions involved produced an amplification, until the present value of about 10^{-6}G was reached. The simulations carried out by Dolag, Bartelmann and Lesch (1999) indicate that initial magnetic field strengths of 10^{-9} G at z=15 provide an amplification of three orders of magnitude in cluster cores. Therefore if B_0 was $10^{-9} - 10^{-8}$G in filaments at Recombination, the subsequent contractions that undoubtedly took place can account for this amplification very easily, only involving two or three orders of magnitude.

b) If magnetic fields are generated via battery processes similar to Biermann's, in the galactic formation itself, then magnetic fields would only be present inside galaxies or in their close vicinity. However, as mentioned above, strong magnetic fields have been observed in the intracluster and in the intercluster media.

4.4.5 Large scale structure and magnetic fields

Coles (1992) suggested that the failure of CDM models to explain large scale structures could be satisfactorily surmounted if magnetic fields were taken into account. Large scale structures are characterized by a noticeable regularity and periodicity (Broadhurst et al., 1990; Einasto et al. 1994; Tucker et al. 1997; Landy et al. 1996; Quashnock et al. 1996; Atrio-Barandela et al. 1997; Tully et al. 1992; Einasto et al. 1997 a,b,c; Retzlaff, 1998; Tadros et al. 1998, Toomet et al., 1999, and others) suggesting a network of filaments.

Battaner, Florido and Jimenez-Vicente (1997), Florido and Battaner (1997), Battaner, Florido and Garcia-Ruiz (1997), Battaner and Florido (1998) and Battaner (1998) have theoretically analyzed the influence of magnetic fields on the large scale structure along the radiation dominated universe and their conclusions may be summarized as follows:

a) Preexisting magnetic fields are able to produce anisotropic density inhomogeneities in the photon fluid and local metric perturbations. In particular, they are able to produce filamentary structures in the distribution of the energy density.

b) Particularly interesting are those filaments larger than about $\sim$10 Mpc, because they have no problems with magnetic diffusion (as mentioned above), because their evolution is more predictable and because they can be observed today. In fact these radiative and gravitational potential filaments were the sites where baryons, or any other dark matter component, collapsed at Recombination, forming the illuminated supercluster filaments that are observed today as elements of the large scale structure. Non linear effects have very much distorted the pre-Recombination structures, as well as the larger ones, though to a much lesser extent, as $\delta\rho/\rho$ remains low. Therefore, these pre-Recombination radiative filaments should be identifiable today.

c) The orders of magnitude of these magnetic fields are equivalent to present $B_0 \sim 10^{-8} - 10^{-9}$G. If they were much lower, they would have no influence on the larger scale structure. If they were much higher, the formation of the galaxy would have taken place much earlier.

d) The filament network, if magnetic in origin, must be subject to some magnetic restrictions. The simplest lattice matching these restrictions is an "egg-carton" network, formed by octahedra joining at their vertexes. This "egg-carton" universe would have larger amounts of matter along the edges of the octahedra, which would be the sites of the superclusters. Outside the filaments there would be large voids, devoid not only of baryons but also of magnetic fields (Fig. 21). Magnetic field lines would be concentrated in the filaments, with their directions being coincident with those of the filaments.

These theoretical speculations are compatible with present observations of the large scale structure as delineated by the distribution of superclusters. It is easy to identify at least four of these giant octahedra in real data, which comprise observational support for the egg-carton universe. Two of them, those which are closest and therefore most unambiguously identified, are reproduced in Fig. 22. Nearly all the important superclusters in the catalogue by Einasto et al (1997), as well as nearly all the important voids in the catalogue by Einasto et al. (1994) can be located within the octahedron structure. This web is slightly distorted by the presence of the very massive Piscis-Cetus supercluster in one of the filaments.

The magnetic origin of structures at very large scales alleviates the old problem encountered by CDM models which predict too little structure at large scales (see, for instance, the reviews by Bertschinger, 1998 and Ostriker, 1993).

A fractal nature could be compatible with the octahedron web, in agreement with the identification of fractals by Lindner et al. (1996) from the observational point of view. There could be sub-octahedra within octahedra, at least in a limited range of length scales. The simplest possibility is reproduced in Fig. 23 in which 7 small octahedra contacting at their vertexes have their egg-carton structure within a large octahedron, the ratio of large/small octahedron size being equal to 3. The fractal dimension becomes quantified, with 1.77 and 2 being the most plausible values. The scale of the fractal structure would range from 150 Mpc, i.e. slightly lower than the deepest surveys, down to about 10 Mpc (in agreement with Lindner et al), as shorter scale magnetic fields would have been destroyed by the resistive radiation dominated universe. Whether the fractal egg-carton structure continues indefinitely for larger scales as suggested by Sylos Labini et al. (1998) and others, remains an open question, but Battaner (1998) proposed this structure under the adoption of the Homogeneity Cosmological Principle at large enough scales.

The absence of a relation between Faraday rotation and redshift of quasars indicates that a widespread cosmological aligned magnetic field must be $B_0 < 10^{-11}$G (Lesch and Chiba, 1997; Kronberg, 1994; Rees and Reinhardt, 1972; Kronberg and Simard-Normandin, 1976; Vallée, 1983, 1990). However, the distribution of large scale magnetic fields is probably very far from homogeneous. Not only $<\vec{B}>=0$, but $<B^2>$, even if not vanishing, is far from homogeneous. Instead, we are interested in the peak values to be found in the matter filaments, in which case this limit should be increased by a large factor, even if it is very low in voids, i.e. in the largest fraction of the volume of the Universe.

There are other upper limits that should be increased by this factor too, if we are interested in the field strength within the filaments: for instance, $B_0 < 10^{-7}$G, taking into account the ^4He abundance (Greenstein, 1969; Zeldovich and Novikov, 1975; Barrow, 1976; Cheng, Schramm and Truran, 1994; Matese and O'Connell, 1970; Grasso and Rubinstein, 1995, 1996 and others), $B_0 < 4 \times 10^{-9}$G, taking into account the neutrino spin flip (though very much depending on the mass of all neutrinos) (Shapiro and Wasserman, 1981; Enqvist et al., 1993), and $B_0 < 4 \times 10^{-9}$G, based on the CMB isotropy (Lesch and Chiba, 1997; Barrow, 1976; Barrow, Ferreira and Silk, 1997).

Observations of the distribution and orientation of warps of galactic disks, under the interpretation that these warps are produced by intergalactic magnetic fields (Battaner et al. 1991; Battaner, Florido and Sanchez-Saavedra, 1990; Zurita and Battaner, 1997) show coherence regions of about 25 Mpc, though Vallée (1991) did not confirm this coherent orientation. In any case, the volume of observed galaxies is too small compared with the scales we are now considering.

The improved sensitivity of experiments intended to measure the CMB would permit us to gather direct information about large scale magnetic fields (Magueijo, 1994; Kosowsky and Loeb, 1997; Adams et al., 1996, and others).Ultrahigh energy cosmic rays can also observe valuable information (e.g. Lee, Olinto and Siegl, 1995; Stanev et al., 1995; Stanev, 1997 and others), as well as the delay in the arrival of the energetic TeV-γ-rays with respect to the low-energy-γ-rays (Plaga, 1995).

4.4.6 A tentative history of cosmological magnetic fields

Despite the large number of theories about the birth, evolution and present distribution of magnetic fields, some general picture seems to emerge and could be summarized as follows.

Magnetic fields were created at Inflation, as predicted and explained in the superstring theory, when the horizon was nearly independent of the cosmological scale factor. Small scale fields were washed out during the resistive radiation dominated universe, but large scale fields, larger than the horizon during the large time interval between Inflation and Recombination, escaped from magnetic diffusion and reentered as subhorizon scale fields.

Along the radiation dominated universe, magnetic flux tubes produced metric perturbations that generated filamentary concentrations of photons and other matter (including dark) components. Small scale radiative filaments, if they were actually formed, were dissipated by photon diffusion mechanisms for masses lower than the Silk mass. Similarly, small scale fields originated by phase transition were dissipated by magnetic diffusion just after Annihilation (and probably also by photon diffusion in the so called Acoustic era, just before Recombination). Large scale radiative baryonic filaments, i.e. larger than the horizon along the Radiative era, survived and reached the Recombination epoch. By the decoupling of photons, dark matter and baryons remained concentrated in the filaments. Matter filaments inheriting the properties of primordial magnetic structures formed a quasi-crystal network mainly consisting of octahedra contacting at their vertexes, reminiscent of an egg-carton topology. Magnetic fields were concentrated into the filaments and conserved their direction. Parameters defining these filaments would be (in equivalent-to-present units): length: $\sim$100 Mpc; width $\sim$10 Mpc; strength $10^{-9} - 10^{-8}$G, at the Recombination epoch, but also existing fractal substructures.

After Recombination, non-linear contractions leading to superclusters, clusters and galaxies corrupted and deformed the initial sharper filaments, becoming clumpy but conserving the large scale alignment. These collapses amplified the magnetic field strength from $\sim 3 \times 10^{-9}$G to $\sim 10^{-6}$G, and galaxies therefore formed out of a microgauss magnetized medium. From the early stages, magnetic fields played an important role in the dynamics of galaxies, mainly in the outermost disk, where they became toroidally ordered, initiated a fast rotation, introduced instabilities into the disk and produced an escape of gas; they were also in part ejected together with the gas.

5 Common halos

Most galaxies are in more or less large clusters. In the analytic, hierarchical CDM scenario, halos are the result of the merging of smaller, less massive, denser, previously formed halos. Once the new large halo is formed, violent relaxation erases any internal substructure, and therefore halos within halos should not be expected from this type of model. As an exception, visible galaxies may survive the merging process, and therefore we might expect to find several visible galaxies in a halo. High resolution N-body simulations have, however, been able to resolve some sub-structures, or subhalos, within dark matter halos (Colin et al. 1999; Benson et al. 1999 and others) even if tidal disruption, spatial exclusion of subhalos, dynamical friction and other effects complicate the global picture. In view of these difficulties and given the number of free parameters inherent to these calculations, let us keep the classical scenario in which: a) no subhalos exist within a halo; b) several visible galaxies may reside in the same halo; c) a halo can have no visible galaxy; d) no visible galaxy can exist outside a halo. This picture is fully compatible with the essentials of hierarchical CDM models, even if the above mentioned particular models keep track of subhalos. The purpose of this argument is to comment on the possible picture in which a large percentage of spiral galaxies are embedded in common halos, instead of each having their own.

Common halos could be present in clusters and associations at all levels, from binary systems to rich clusters. The hypothesis of common halos is not new (see the review by Ashman 1992, for instance). Let us briefly consider the different systems:

a) **Dwarf irregular satellites around a bright galaxy**. Following White & Rees (1978), when the halos of the first small galaxies are disrupted to form bigger units, the residual gas may again be able to cool and collapse to form a larger central galaxy. The model naturally predicts the existence of small satellites around big galaxies. Therefore, this and some subsequent models implicitly assume that the satellites have no halo of their own, but are instead in the halo of the bright galaxy, even if observations seem to indicate that these satellites are DM rich (e.g. Ashman 1992).

b) **Binary galaxies**. In the well-known paper by Kahn & Woltjer (1959), it was considered, as an alternative interpretation, that the unseen mass was forming a common envelope. van Moorsel (1987), observing HI in binaries, suggested that the data were consistent with a common dark matter envelope surrounding the pair system. Charlton & Salpeter (1991) concluded that extremely extended halos, with radii of around 1 Mpc, were present in their sample of binaries. This could also be the case of the M31-Milky Way pair; both could lie in a common halo that has arisen from the mergers of the early smaller halos of the two galaxies.

c) **The Local Group**. If the 35 galaxies, or more, members of the Local Group conserved their own halos, there could be a problem with available volume. To calculate the filling factor, i.e. the volume of the halos of the 35 galaxies divided by the total volume of the Local Group, we face the problem that we do not actually know the individual volumes. But for an exploratory calculation, we may assume that all halos have the same volume, irrespective of type and luminosity. To justify this assumption, let us consider that R_{200} is the halo size. From its definition (the radius enclosing a sphere with mean density 200 times the critical density) it is easily deduced that $R_{200} \propto M_{200}^{1/3}$ for all galaxies. A relation should exist between M_{200} (the mass of a sphere with radius R_{200}, which can be taken as the mass of the halo) and the luminosity L, an observational quantity. Salucci & Persic (1997) give $M_{200} \propto L^{0.5}$, in which case $R_{200} \propto L^{0.17}$, i.e. R_{200} is "nearly" independent of the luminosity. White et al. (1983) and Ashman (1992) propose $M_{200}/L \propto L^{-3/4}$, in which case $R_{200} \propto L^{0.08}$; in this case, the exponent (0.08) is so small that the adoption of constant R_{200} for all galaxies is a good first approximation. Let us take $R_{200} \sim 250$kpc as a typical value. Let us adopt the zero-velocity surface radius (1.18 Mpc; van dem Bergh 1999) as the radius of the Local Group halo. The filling factor obtained is then 0.33. This figure is so high that individual halos would be in contact, and eventually merge. Therefore, a picture more in consonance with the theory is that there is only one large previously formed common halo. This rough calculation just considers the most optimistic situation. The filling factor would be higher if the Local Group were non spherical as suggested by Karachentsev (1996) and if there were many more galaxies belonging to the Local Group. Discoveries of new members have recently been reported and many low surface brightness galaxies would not have been detected if they had not been in the close vecinity of the Milky Way. Moreover, consider that in a sphere of 500 kpc around the Milky Way there are 11 galaxies and around M31 there are 15 galaxies. Under the assumption that each galaxy has its own halo of about 200 kpc, we obtain filling factors much higher than unity. Another observation suggesting that the Local Group has a common halo is the observation that the high-velocity clouds have their kinematic centre in the Local Group barycentre (López-Corredoira, Beckman & Casuso 1999).

d) **Small compact groups**. The evidence and necessity of common halos is specially clear in the case of Hicson Compact Groups (HCG; Hickson 1982;

Mamon 1995). HCG contain few galaxies, four or slightly more, and are very compact, with the intergalactic distance and the whole apparent size of the group being much smaller than the size of typical halos. From the dynamical point of view, given the small velocity dispersion, the system would collapse in less than 10^9 years, after which the members would merge and form a large elliptical (Barnes 1989; Diaferio, Geller & Ramella 1994 and others) but in fact they are noticeably stable and there are few signs of interaction and merging. These facts led Athanassoula, Makino & Bosma (1996) to assume a massive, not excessively concentrated common DM halo. Gómez-Flechoso & Domínguez-Tenreiro (1997) included a common DM halo in their N-body simulations to stabilize the groups. Common envelope material is found in X-rays (Ponman & Bertram 1993) and atomic hydrogen (Verdes-Montenegro et al. 1997 and references therein). In HCG 49, Verdes-Montenegro et al. (1999) showed that the HI common envelope is rotating with a highly symmetrical pattern, following a large-scale potential that is not due to any particular galaxy member. Perea et al. (1999) have studied faint satellite galaxies at large distances from the members but belonging to the HCG. They found that the common halo is about four times more massive than the galaxy members. We therefore conclude that a great deal of evidence clearly indicates that HCG are embedded in large common halos.

e) **Rich clusters**. White & Rees (1978), Navarro, Frenk & White (1996) and many other theoretical models had as objectives the obtention of halos with different sizes, with rich clusters being the largest considered. Clearly, rich clusters could be the best example of visible galaxies moving in a large DM cluster, from the point of view of hierarchical CDM scenarios.

Therefore, the hypothesis of a common halo, as opposed to individual halos, is compatible with observations of galaxy pairs, almost essential for groups like the Local Group, compelling for compact groups and tempting for rich clusters. It is also qualitatively coherent with the scenario assumed by hierarchical CDM. In analytic and semianalytic models a large common halo is assumed to be virialized; the violent relaxation following the successive merging processes would destroy any DM substructure, though visible galaxies could remain indigest. Then isolated spirals would not possess a DM halo. Some numerical calculations have not found any subhalos within halos (e.g. Katz & White 1993; Summers, Davis & Evrard 1995) giving rise to the so-called "overmerging" problem.

Benson et al. (1999) obtain many small halos containing no visible galaxy, which could be due to feedback from supernovae, which prevents efficient galaxy formation. Though they obtain that the mass-to-light relation has a minimum for about $10^{12} M_\odot$, the number of visible galaxies in a halo greatly increases with halo mass (at least for their ΛCDM model) indicating that large halos are indeed common halos of many galaxies. The number of visible galaxies with blue absolute magnitude brighter than about -19.5 per halo is statistically lower than unity, but this number probably increases when fainter galaxies are considered in the results of this model. The Local Group has only two galaxies brighter than -19.5.

Moore et al. (1999) and others find a DM substructure in DM halos. Following this calculation, the Milky Way would have about 500 satellites with mass $\geq 10^8 M_\odot$, and therefore mechanisms avoiding stellar formation within so many small halos would be required, which has been discussed by Moore et al. and references therein. Internal mechanisms do not seem to be responsible: if gas is lost by star-bursts and winds in a first stage of star formation, it should be explained why galaxies outside clusters have rotationally supported disks. Moreover, the strongest star-bursts observed in nearby dwarf galaxies are insufficient. These authors also discuss the difficulties inherent in forming and maintaining disks in the presence of large amounts of substructure, as disk and small halo interactions will frequently heat disks and produce ellipticals.

Galaxies could have a very different behaviour depending on their position in a cluster. Whitemore, Forbes and Rubin (1988) found a relation between the gradient of rotation curves and the location in the cluster. Verheijen (1977) found an anomalous behaviour of rotation curves of galaxies belonging to the Ursa Major cluster. Rubin, Waterman and Kenney (1999) have found many galaxies with kinematic disturbances in the Virgo cluster, but tidal effects and accretion events can explain the observed disturbances. Individual dynamic studies of disks in clusters are difficult to interpret.

We could conclude that the existence of common halos and the non-existence of individual sub-halos are suggested both by the observations and by the theory. Following this picture then, spiral galaxies would have no halo, but rather move inside halos, orbiting off-centre in less dense and more homogeneous DM environments.

Truly isolated spiral galaxies would have their own halo but these are exceptional. van dem Bergh (1999) estimated that half of all galaxies in the Universe are situated in small clusters such as the Local Group. Soneira & Peebles (1977) estimated at 15% the number of isolated galaxies. Tully & Fisher (1978) even claimed that there is no evidence for a significant number of field galaxies.

The situation would be similar for late type irregulars and for dwarf spheroidal galaxies. However, these conclusions would not serve for the DM content of elliptical galaxies. In a rich cluster, if we assume that the centres of the DM halo and of the galaxy distribution coincide, at least the giant cD ellipticals at the centre would have large quantities of DM, with the cD galaxy and the DM also being coincident. Giant ellipticals, like M87, have been considered to possess very large amounts of dark matter since Fabricant, Lecar & Gorenstein (1980) and Binney & Cowie (1981) analyzed their X-ray emission. Indeed these authors noticed that these large quantities of DM encountered could belong to the cluster itself rather than to the galaxy. Huchra & Brodie (1987) showed that the dynamics of globular clusters around M87 supported the huge mass found from X-ray observations, of the order of $10^{13} M_\odot$. It is unclear whether this conclusion about cD galaxies would also apply for normal ellipticals. In some cases, the debris from a merger of spirals could fall into the halo centre. The difference between spirals (and irregulars) and cD ellipticals would be that the former lie well outside the halo's centre while the latter coincide with it.

Therefore, we can summarize the present crossroads of the problem of rotation curves of spiral galaxies by emphasizing that, if we accept the hypothesis of common virialized halos, with no substructure, for all types of clustered visible galaxies, then there are only two alternatives:

Either hierarchical CDM models are wrong, for instance, DM is baryonic (e.g. de Paolis et al. 1997; Pfenniger & Combes 1994), in which case we would need a theory of galaxy formation.

Or they are basically valid, in which case, another explanation of the rotation curve is needed. For instance, forces other than gravitation could be involved, so that models of galaxy formation would have no "responsibility" in explaining the rotation curve. We should take into account the magnetic hypothesis (Nelson 1988; Battaner et al. 1992, Battaner & Florido 1995, Battaner, Lesch & Florido 1998). Given the success of current theoretical CDM hierarchical models in other related topics, we favour this latter possibility.

6 Conclusions

a) Standard interpretation of rotation curves.

There is a general consensus about the history of galaxy formation and the establishment of the rotation curve of spirals. This standard history could be summarized as follows:

At Inflation, quantum mechanical fluctuations were generated and then survived until the epoch of Recombination. The Universe then became CDM dominated with a baryonic component as a minor constituent. By gravitational collapse the primordial fluctuation that evolved gave rise to small DM halos. Adjacent halos merged to produce larger halos and this merging process has continued until the present. A complete hierarchy of CDM halos has been produced, those produced later being larger and the size being limited by the finite time of the Universe.

Once a new halo is formed by merging, violent relaxation effects destroy part of the previous DM substructure, but not completely, leaving some CDM subhalos still identifiable. After that, baryons cool and concentrate at the CDM centre at any size of the hierarchy. Baryon concentrates then form galaxies and shine. Larger CDM halos, produced later, also produce larger visible galaxies.

Hierarchical CDM models predict universal halo density distribution profiles, the so called NFW profiles, irrespective of their size and position in the hierarchy, following as R^{-1} in the inner region and as R^{-3} in the outer one. The CDM halo density profiles decide the rotation curve of the visible galaxies which are small bright indicators of large and massive CDM halos.

Subhalos within a halo are possible and would correspond to the existence of small ancient visible satellite galaxies orbiting around a large galaxy or to normal galaxies in a large cluster. Some CDM subhalos were destroyed in the merging process but their visible baryonic aggregates, because of their high density, were able to survive. Visible galaxies also merge. The merging of two or more former disk galaxies produces a larger elliptical.

Some aspects in this short account of the long history are better known than others. If we restrict ourselves to the rotation of spiral galaxies, there are some problems that remain unclear or are insufficiently explained. We prefer now to select problems rather than to emphasize successes. Among the outstanding problems let us highlight the following:

- Theorists themselves are unhappy with the rotation curves obtained; in particular the Tully-Fisher relation is unsatisfactorily explained.

- It is not clear if the visible galaxies should now be at the centres of their "own" halos or if they lie off-centre in common halos shared with other galaxies. The degree of CDM substructure predicted by different authors varies. In the first pioneering steps it was suggested that dwarf satellites would be in the halo of the larger primary galaxy, having no smaller halos of their own. Observations, however, and new theoretical model developments, indicate that dwarf satellites not only have their own halos, but also that the dark/visible matter ratios are much larger.

- The rotation curves observed can be fitted to the so called Universal Rotation Curves, but their density profiles do not coincide at all with the theoretical density profiles. The universal rotation curves have a core, i.e. a region in which the density is more or less constant or slightly decreasing as R, and in the periphery they decrease as $R^{-3/2}$.

- The so called "halo-disk conspiracy", i.e. why the disk and halo dominated regions have a featureless flat transition, is not completely answered. The fact that some galaxies have rising or declining curves does not explain the conspiracy problem in those galaxies that do possess a flat curve. The adiabatic compression of the inner halo due to the disk formation establishes a halo-disk relation that alleviates the conspiracy, but much work is still needed to model this interaction.

- Both universal rotation curves and universal halo density distributions shed some light on the absence of correlation between the orbital velocities of satellite galaxies and rotation velocity of the primary galaxies (or equivalently their luminosities, as Tully-Fisher relates both quantities). But both velocities should be determined by the halo and should partially correlate.

The so called Bosma relation, even if some authors have their doubts about its validity, indicates a relation between the circular velocities of both the dark halo and the gas. This relation is not only a general one, but in some galaxies, small scale changes are present in the rotation curve and in the gas distribution. This seems paradoxical, particularly if we consider that the dark matter is more or less spherically distributed and the gas lies in a disk.

Clearly, other authors would discuss other points that are still obscure and others would have preferred to focus on the agreements with observational facts, which are certainly encouraging and suggest that the basic scenario has been firmly established. There are, however, many other possible histories: we have seen that some authors consider dark matter to be baryonic and even that it is in the disk. These theories have considerable merit, especially because they must be developed against the general flow of ideas, and because they explain some observational facts in a simpler way.

Let us just outline another alternative history, rather different from the above mentioned standard one, if the word "standard" can be properly assigned to any of the present scenarios.

b) The magnetic interpretation of the rotation curve.

At inflation, quantum mechanical fluctuations were generated not only in the energy density distribution but also created a non uniform distribution of magnetic fields. Magnetic flux tubes, probably interconnected with other tubes forming a network, conserved their shapes during the radiation dominated era, but the strength decreased as the expansion proceeded. Finite conductivity effects destroyed the small scale magnetic field structures, but those that were large enough, larger than about 50 Mpc (comoving) survived, producing filaments in the energy density distribution -probably also sheets- which after Recombination became dark matter filaments (with baryons as a minor constituent), more than 100 Mpc long.

The scenario provided by hierarchical CDM models, assumed in the "standard" history previously summarized, is fully assumed here too, with the only exception that mergers and non-linear evolution took place inside the large density filaments and not in the voids in between. The heating produced by shocks in the merging events also affected the magnetic fields, which became disordered and amplified. Individual halos belonging to a visible galaxy at its centre were formed in the first generations of halos, but subsequent mergers produced common halos shared by satellites or groups of galaxies. Pairs, satellites, poor clusters and rich clusters developed their superhalos with no dark matter substructure. Only exceptional truly isolated visible counterparts would retain their own halo against merging or were the result of merging of the visible galaxies in a previous common halo.

The fact that hierarchical mergers only took place within the filaments and not in the large voids, assumed here to be of primordial origin, could alleviate a problem encountered in CDM hierarchical models: that of overproduction of halos. Furthermore, these models predict too little structure at scales larger than about 40 Mpc, a problem that was detected in the pioneering simulations.

The disk was formed out of magnetized gas and maintained a magnetic pressure equilibrium with the region outside the visible galaxy, frozen-in in the low-density uncondensed gas lying in the common halo. Equilibrium was possible because of the frequent outbursts of disk material and magnetic fields due to violent star formation events, as observed for instance in M82 and in other galaxies like ours. This magnetic field acquired a toroidal distribution, due to the differential rotation, which was able to generate a centripetal force which produced a higher rotation in the periphery of disks.

Magnetic fields responsible for the high rotation velocity also rendered the disk thicker, facilitating the fountain effect and escape. Magnetic fields would act inwards in the radial direction and outwards in the vertical direction. The escape from the disk and even from the galaxy would be a more important effect in dwarf irregulars, which indeed present larger outbursts of material; they are gas rich, and therefore have greater ability to amplify magnetic fields. Under this interpretation, irregulars are not DM rich galaxies, but magnetic field rich galaxies, because they are gas rich galaxies.

One question naturally arises in this scenario. What is the DM content of the other galaxies, particularly in ellipticals? Suppose a rich cluster which has a giant cD elliptical at the centre. It is evident that cD galaxies are then also at the centre of the cluster superhalo and therefore, they are beated in a region with large amounts of dark matter. Therefore, cD galaxies should have large amounts of DM, as seems to be the case in M87. It is less clear what is the situation with normal ellipticals. Therefore, we merely propose that giant ellipticals at the centre of large halos would have large quantities of DM; spirals, lying far from the giant common cluster halo centre would not posses dark matter halos. Even if they were embedded in DM, this would be more homogeneously distributed around the spiral, and hence it would not have such a decisive influence on the rotation curve. The DM halo potential could produce warps as a tidal effect.

In the "standard" history, we discussed some current problems and disagreements between theory and observations. In this alternative scenario, we comment on their advantages. In this paper we have reviewed a model that numerically accounts for the basic facts. From a qualitative point of view, without any precise developments, let us also consider:

- Galaxies with more gas would produce, and be subject to, higher magnetic fields, and precisely these galaxies were considered to have rotation curves with a greater discrepancy from the curve expected from the gravitation produced by disk and bulge.

- The Bosma relation, establishing a connection between gas and DM (which in this picture should be expressed as a relation between gas and magnetic fields) would be obvious.

- There would be no conspiracy problems, as the magnetic fields and gravitation forces ratio would be progressive and continuously increasing for increasing radii.

- The problems arising from the lack of correlations in binary galaxies would naturally disappear. The velocity observed at the higher radius where the signal is detected would be the result of internal magnetic fields, clearly unrelated to any orbital velocity, whether or not the pair lies in a common halo.

- The agreement with theoretical hierarchical CDM models is better, as these models have "no responsibility" to directly explain rotation curves, unless they include magnetic fields. For instance, the theoretical prediction that irregulars orbiting a bright galaxy (like ours) would be embedded in its halo and would not have their own halo, would not contradict the standard interpretation of observations that irregulars are particularly dark matter dominated.

c) Other alternatives.

Theories assuming galactic dark matter to be undetected gas must eventually answer two basic questions. How have these galaxies formed? and, if $\Omega_M \sim 0.3$ and $\Omega_B \sim 0.03$, where is the non-baryonic dark matter? (could the answer to the second question be the existence of common cluster halos?).

Theories proposing a modification of Newton's Second Law should clarify whether we should also reject General Relativity. Modifying Newtonian Mechanics is a tolerable sacrifice, but physics would probably require more solid proof before abandoning General Relativity.

Summarizing this summary, we are beginning to understand galaxy formation, the nature and distribution of dark matter in galaxies and rotation in what could be called the standard scenario. But there are other interesting alternatives that should not be disregarded without an intense debate. MOND is one of them. Gaseous dark matter is another. The magnetic alternative is not frontally opposed to CDM hierarchical scenarios, but is, rather, complementary. Only secondary phenomena are in clear contradiction. It is unrealistic to attempt to deal with rotation curves while ignoring magnetic fields. This could constitute a particular flaw in the standard model for rotation curves.

If after this review, the topic of galactic dark matter is less clear, we will have accomplished our mission.

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The literature about dark matter is really copious. Some reviews have been particularly useful in preparing this one, such as those by Trimble (1982) "Existence of dark matter in the Universe", Ashman (1992) "Dark matter in galaxies", Bosma (1998) "Dark matter in disk galaxies", Fich and Tremaine (1991) "The mass of the Galaxy" and those appeared in the book "Dark and visible matter in galaxies" (1997), as those by Salucci and Persic (1997) "Dark matter halos around galaxies", Pfenniger (1997) "Galactic Dynamics and the nature of dark matter", Frenk et al. (1997) "Numerical and analytical modelling of galaxy formation and evolution", Navarro (1997) "Cosmological constraints from rotation curves of disk galaxies" and others. The whole book was decisive to prepare this review, as it is a recent complete and updated compilation by many workers on the field. The PhD theses of Begeman (1987) "HI rotation curves of spiral galaxies", Broeils (1992) "Dark and visible matter in spiral galaxies" and Swaters (1999) "Dark matter in late-type dwarf irregulars" were also special sources of information. For the section devoted to magnetic fields, some reviews were specially detachable: those by Kronberg (1994) "Extragalactic magnetic fields", Lesch and Chiba (1997) "On the origin and evolution of galactic magnetic fields", and Vallée (1997) "Observations of the magnetic fields inside and outside the Milky Way", starting with globules (~ 1 parsec), filaments, clouds, superbubbles, spiral arms, galaxies, superclusters, and ending with the cosmological Universe's Background Surface (at ~ 8 Teraparsec).

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Figure captions

Figure 1.- Upper panel: The luminosity profile of NGC 2403 observed by Wevers, van der Kruit and Allen (1986). Lower panel: The observed rotation curve of NGC 2403 (dots) and the rotation curves of the individual mass components (lines). From Begeman (1987) PhD thesis.

Figure 2.- A position-velocity map along the major axis at a position angle of 21^o. The filled circles show the adopted rotation curve of NGC 1560. The cross in the lower right corner indicates the angular and velocity resolutions. From Broeils (1992). Courtesy of Astronomy and Astrophysics.

Figure 3.- The observed rotation curve and disk-halo models. The top panel shows a halo added to the maximum disk; the middle panel shows the "maximum halo", with almost no mass in the stellar disk ($M/L_B = 0.1$). The bottom panel shows the "best fit" disk-halo model. The dotted and dashed lines indicate the gas and stellar disks. The dot-dashed line shows the rotational velocities of the halo; the full line is the contribution of all the components. From Broeils (1992). Courtesy of Astronomy and Astrophysics.

Figure 4.- Left: Upper panel: The luminosity profile of NGC 5033 observed by Kent (1986). Lower panel: The observed rotation curve of NGC 5033 (dots) and the rotation curves of the individual mass components (lines). Right: Upper panel: The luminosity profile of NGC 5371 observed by Wevers, van der Kruit and Allen (1986) and its decomposition into a bulge and disk (lines). Lower panel: The observed rotation curves of NGC 5371 (dots) and the rotation curves of the individual mass components (lines). From Begeman (1987) PhD thesis.

Figure 5.- Comparison of stellar and gaseous rotation curves by Vega-Beltrán (1999). Upper window: R-band image. V-window: rotation velocities. The other windows represent the velocity dispersion and the third and fourth orders Gauss-Hermite of the line-of-sight velocity distribution of the stars. From Vega-Beltrán (1999) PhD thesis.

Figure 6.- Same as for figure 9. From Vega-Beltrán (1999) PhD thesis.

Figure 7.- The Milky Way rotation curve in (a) a logarithmic scale by Sofue et al. (1999) as compared with the linear scale rotation curve (b). Courtesy of American Astronomical Society.

Figure 8.- The observed rotation curve of NGC 2460 together with a maximum-disk+halo model. From Broeils (1992) PhD thesis.

Figure 9.- Logarithmic slope between two and three disk scale lengths $S^h_{(2.3)}$ versus absolute R-band magnitude M_R. Filled circles correspond to late-type galaxies in a high quality rotation curve sample, open circles represent dwarfs in a lower quality rotation curve sample, open triangles are galaxies in Verheijen's (1997) Ursa Major sample, filled triangles represent the galaxies from various sources presented in Broeils (1992). From Swaters (1999) PhD thesis.

Figure 10.- The Tully-Fisher relation for spiral and late-type dwarf galaxies. Symbol coding as in Fig. 13. From Swaters (1999) PhD thesis.

Figure 11.- Overall rotation curves of the Galaxy for $\theta_0 = 220 kms^{-1}$ (filled circles), $\theta_0 = 200 kms^{-1}$ (open circles), and $\theta_0 = 180 kms^{-1}$ (open triangles).

The data for the inner rotation curve were taken from Fich et al. (1989). The outer rotation curve are those obtained by Merrifield's method. The error bars are indicated only for $\theta_0 = 220kms^{-1}$, and are almost the same for the three cases. From Honma and Sofue (1997). Courtesy of the Astronomical Society of Japan.

Figure 12.- Observed rotation curve of NGC 404, uncorrected for inclination; crosses: observations, solid squares: Keplerian decline. From del Río et al. (1999). Courtesy of the Astronomical Society of the Pacific.

Figure 13.- Different possibilities to understand the negative radial velocity of M31.

Figure 14.- The evolution of Jean's mass as a function of a (the cosmological scale factor taking its present value as unit, i.e. $a = R/R_0$). Points over the curve correspond to unstable situations leading to gravitational collapse. An inhomogeneity as the Milky Way, with a rest mass of about $10^{12} M_\odot$ was unstable until a slightly greater than 10^{-6}. Then, it underwent acoustic oscillations until the epoch of Recombination, when it become unstable again. Adopted from Battaner (1996).

Figure 15.- The evolution of the relative overdensity of an inhomogeneity cloud as a function of $a = R/R_0$, being R the cosmological scale factor and R_0 its present value. Adopted from Battaner (1996).

Figure 16.- The redshift distribution of galaxies with magnitudes in the range 22.5 $<$ B $<$ 24.0. The solid histogram shows the data of Glazebrook et al. (1995), while the dash histogram shows the (more complete) data of Cowie et al. (1996). The lines show the predictions of the model of Cole et al. (1994) for a Scalo IMF (solid line) and a Miller-Scalo IMF (dotted line). From Frenk et al. (1997). Courtesy of the Astronomical Society of the Pacific.

Figure 17.- Present-day B-band luminosity functions in a model with [Ω_0 =1, $\Lambda_0 = 0$, h=0.5, $\sigma_8 = 0.67$] (solid lines) and another with [Ω_0 =0.3, $\Lambda_0 = 0.7$, h=0.6, $\sigma_8 = 0.97$] (dotted lines). In each case, the extended curve shows the luminosity function of the galaxy population as a whole. The shorter curve shows the luminosity function of galaxies that contained at least one progenitor satisfying the selection criteria for Lyman-break galaxies at high redshift in the study of Steidel et al. (1995). The data points show observational determinations of the luminosity function from Loveday et al (1992) and Marzke et al. (1994). From Baugh et al. (1998). Courtesy of the Astronomical Society of the Pacific.

Figure 18.- NFW circular velocity profiles for different values of the concentration. The circular velocity at the virial radius, V_{200}, is chosen to match the rotation speed in the outskirts of the galaxy, and varies from 100 to 130 km s^{-1} from top to bottom. As illustrated in this figure for NGC 3198, most galaxy rotation curves can be adequately parametrized. The best fit value of the c parameter ($c_{obs} \approx$26 in this case) provides a quantitative measure of the shape of the rotation curve. Low values of c_{obs} denote slowly rising curves, while high values of c_{obs} indicate steeply rising, flat or declining rotation curves.

Furthermore, c_{obs} constitutes a firm upper limit to the concentration of the halo, since the fit neglects the contribution of the luminous component. From Navarro (1999).

Figure 19.- Top-left: Rotation curve shape characterized by the parameter c_{obs}, the value of the concentration deduced by the observations of the rotation curve, versus effective surface brightness ($\Sigma_{eff} = L/\pi R^2_{disk}$). Top-right: V_{200} versus V_{rot}, commented in text. In all four plots: dotted lines assume that $V_{rot} = V_{200}$ and only $(M/L)_{disk}$ is adjusted to match the Tully-Fisher relation. Short-dashed lines assume that $(M/L)_{disk} = hM_{\odot}/L_{\odot}$ in all galaxies; only V_{200} is varied to match the Tully-Fisher relation. Solid lines correspond to varying both $(M/L)_{disk}$ and V_{200} so as to match the Tully-Fisher relation and the $\Sigma_{eff} - c_{obs}$ correlation. From Navarro (1999).

Figure 20.- Top left: Alfvén's speed in km s^{-1} for three values of the free parameter k (10^{-9}, upper curve; 3×10^{-9}, middle curve; 10^{-8}cm^{-1}s, lower curve). Top right: Magnetic field strength in gauss. Botton left: The flaring function Z(r) in kpc, for three different values of the free parameter k defined in the text (10^{-9}, upper curve; 3×10^{-9}, middle curve; 10^{-8}cm^{-1}s, lower curve). Botton right: Density profiles for the value of the free parameter $k = 3 \times 10^{-9}$cm^{-1}s, for three different values of z=0, 4 and 8 kpc from the galactic plane. From Battaner and Florido (1995). Courtesy of the Royal Astronomical Society.

Figure 21.- Ideal scheme of the egg-carton universe formed with octohedra only contacting at their vertexes. Adopted from Battaner and Florido (1997). Courtesy of Astronomy and Astrophysics.

Figure 22.- The two large octahedra closer to the Milky Way.

Figure 23.- Ideal scheme of the fractal geometry of the octahedron network. In this figure we plot the case of a fractal dimension equal to 1.77. A value of 2 is also an interesting possibility. Adopted from Battaner (1998). Courtesy of Astronomy and Astrophysics.